

Revision Guide

Physics - Unit 3

GCE A Level WJEC

These notes have been authored by experienced teachers and are provided as support to students revising for their GCE A level exams. Though the resources are comprehensive, they may not cover every aspect of the specification and do not represent the depth of knowledge required for each unit of work.

Contents

Page	Section
3	3.1 – Circular motion
6	3.2 - Vibrations
14	3.3 - Kinetic Theory
19	3.4 - Thermal physics
24	3.5 - Nuclear decay
30	3.6 - Nuclear Energy
35	Acknowledgements

Section 3.1 - Circular Motion

Some basic definitions

Frequency, f

The frequency of an oscillating system is defined as the number of oscillations per second, and hence for a rotating system, it is defined as the number of revolutions per second.

The S.I. unit for frequency is the **Hertz, Hz**.

It is also, of course, acceptable to use the unit **per second, s^{-1}** , although for rotating systems it is more convenient to state the number of revolutions per second, or $\text{revs } s^{-1}$.

Example

A drill rotates at 600 rpm (revolutions per minute). What's the frequency of rotation?

$$f = 600 \text{ rpm} = 600 / 60 \text{ revs per second} = 10 \text{ s}^{-1} \text{ or } 10 \text{ Hz}$$

Period, T

The period of an oscillating system is defined as the time taken to complete one complete oscillation.

Similarly, the period of a rotating system is defined as the time taken to complete one revolution.

The S.I. unit for period is therefore the **second, s**.

The relationship between the period and frequency is very simple, as the comparison of their respective units implies:

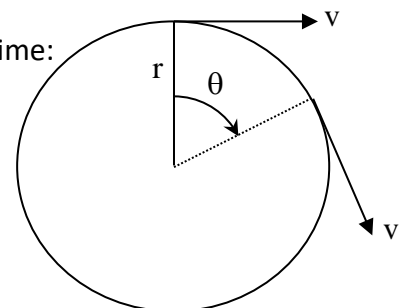
$$f = \frac{1}{T}$$

Angular velocity, ω (Other names : angular speed, angular frequency, pulsance)

If a particle is moving in a circular path at a steady **speed**, then it will rotate through an angle, θ in a time, t .

The angular velocity is defined as the angle swept out by the radius in unit time:

$$\omega = \frac{\theta}{t}$$



Hence the S.I. unit for ω is **$\text{rad } s^{-1}$** .

A specific case : When $\theta = 2\pi$ (i.e. one full circle), then $t = T$ (the period), and hence,

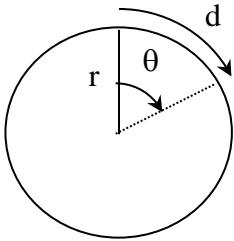
$$\omega = \frac{2\pi}{T}$$

.... and hence...

$$\omega = 2\pi f$$

The radian

The radian is another unit for measuring angles. It is defined as follows:



If a particle moving in a circle moves a distance, d , equal to the radius along the circumference, then the angle rotated through is equal to **one radian**, or **1 rad.**

hence,

$$2\pi \text{ (rad)} = 360 \text{ (}^\circ\text{)}$$

To convert between radians and degrees we must reduce one side of the above relationship to unity, e.g.

Converting degrees to radians $\rightarrow \frac{2\pi}{360} \text{ rad} = 1^\circ$

Converting radians to degrees $\rightarrow 1 \text{ rad} = \frac{360^\circ}{2\pi}$ (about 57.3°)

Example

What's 1.4 radians in degrees? $1 \text{ rad} = 360^\circ / 2\pi \therefore 1.4 \text{ rad} = 1.4 \times (360/2\pi) = 80.2^\circ$

What's 30° in radians? $1^\circ = 2\pi/360 \text{ rad} \therefore 30^\circ = 30 \times (2\pi/360) = 0.52 \text{ rad}$ (or $\pi/6 \text{ rad}$)

Relationship between velocity and angular velocity

The speed of any moving particle is given by the equation,

$$\text{Speed, } v = \text{distance} / \text{time}$$

If we apply this to a particle moving in a circle, we have,

$$v = \frac{\text{arc length}}{\text{time}} = \frac{r\theta}{t} = r\omega \quad (\text{since } \theta/t = \omega)$$

$$v = r\omega$$

where ω = angular velocity

Centripetal acceleration

Newton's first law states that if a particle is at rest it will remain stationary, or if already moving, will continue to move with constant **velocity**, unless acted upon by an external force.

So, for the velocity (which is a vector) to be constant, it is required that the speed is constant **and** that there is **no** change in direction.

Hence, if a particle is moving at a steady speed but following a circular path, then it must be **accelerating**, since the velocity is constantly changing its direction. The direction of this acceleration is always towards the **centre of the circle**. Whatever causes this acceleration, it can always be referred to as the centripetal acceleration, and is calculated thus:

$$a = \frac{v^2}{r}$$

or

$$a = \omega^2 r$$

Centripetal force

Acceleration is only possible if there's a **resultant** force. The acceleration experienced by a body that's rotating can be caused by many different types and combinations of force. However, remember that the resultant force, $\Sigma F = ma$. In this context, this resultant force is called **the centripetal force**, is **always directed towards the centre of the circle**, and since $a = v^2 / r$ or $a = \omega^2 r$

the centripetal force, F_{CP} (or ΣF) = $m a = m v^2 / r$ or $F_{CP} = m a = m \omega^2 r$

Here are two examples of forces combining to create the necessary centripetal force:

1) A roller coaster doing a 'loop the loop'

At the very top of the loop, there are two forces contributing to the centripetal force in this instance – the weight of the roller coaster, W , and the normal reaction from the track, NR , both acting downwards, hence,

$F_{CP} = W + NR$ directed (downwards) towards the centre of the loop

Either ' mv^2/r ' or ' $m\omega^2r$ ' can be substituted for ' F_{CP} '.



2) A child standing on a playground 'merry-go-round'

Although gravity acts on the child, her weight doesn't contribute to the centripetal force since the weight acts vertically downwards. The centripetal force always acts towards the centre of the circle, which in this case, is directed horizontally.

Therefore, the only force acting that causes circular motion is the friction between the child's feet and the surface of the merry-go-round:



$F_{CP} = \text{Friction}$ directed horizontally towards the centre

Example

In the 2nd example above, the maximum friction that the child's feet can generate is 114N. Her mass, m , is 32.5kg. Given that she is 1.72m from the centre, calculate:

- (a) the maximum angular velocity that's possible so that the child remains on the ride,
(b) the minimum rotational period relating to part (a) above.

(a) $F_{CP} = \text{Friction}$, hence, $m \omega_{max}^2 r = 114$

$$\therefore \omega_{max} = \sqrt{114 / (32.5 \times 1.72)} = 1.43 \text{ rad s}^{-1}$$

(b) $T_{min} = 2 \pi / \omega_{max} = 4.4\text{s}$

Section 3.2 - Vibrations

Simple Harmonic Motion (SHM)

There are many types of vibrations but perhaps the most common is that which is known as **simple harmonic motion**. It is important not only because there are many examples of it but also because all other vibrations can be treated as if they are composed of two or more simple harmonic motions.

SHM is defined as follows:

An object is said to be exhibiting SHM if that object moves such that its acceleration is always directed toward a fixed point and is proportional to its distance from the fixed point.

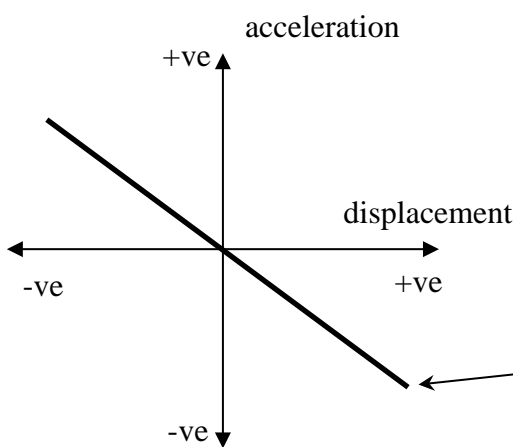
Hence,

$$a = -\omega^2 x$$

where

a	=	acceleration (ms^{-2})
ω	=	angular velocity (rad s^{-1}),
x	=	displacement from equilibrium position (m)

A graph of acceleration against displacement for a particle or system oscillating with SHM would therefore always take the following form:



What is the significance of this graph's gradient?

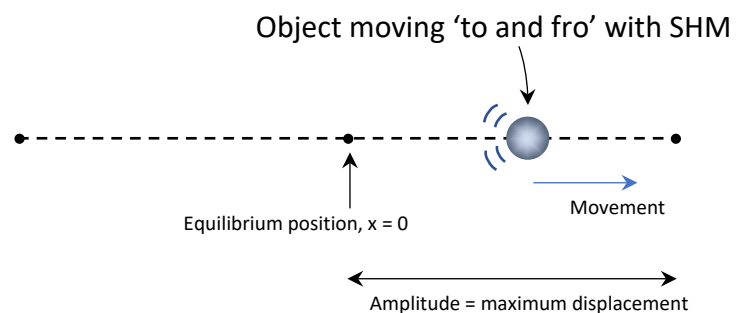
$$a = -\omega^2 x$$

$$y = m x$$

Comparing the 2 equations, it can be seen that,

$$\text{Gradient} = -\omega^2$$

So, as the SHM object moves, its acceleration increases as it goes further from the equilibrium position.



Thus, the acceleration is zero when $x=0$ m and the object is passing through the equilibrium position. The acceleration is a maximum when x is a maximum, i.e. when $x = A$ (where "A" is the amplitude). Hence,

$$a_{\text{max}} = -\omega^2 A$$

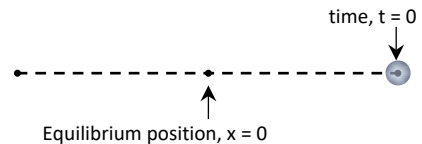
Other equations that describe SHM

The equation for the displacement, x , of the oscillating object at a certain time, t , is:

$$x = A \cos(\omega t)$$

but where does 't' start? Setting $t = 0$, $x = A \cos(0)$, hence, $x = A$. So, the 'stopwatch' starts when the object is at maximum (positive) displacement.

If we set the positive direction to the right in this case, then the 'stopwatch' begins when the particle is at the most extreme right, as shown \rightarrow .



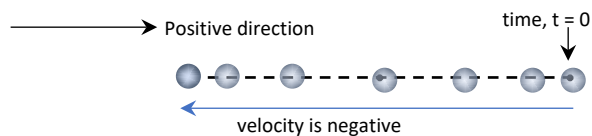
NOTE !!

The displacement is ALWAYS measured from the equilibrium position, regardless of where the 'stopwatch' is started.

The **corresponding** equation for the velocity, v , of the object exhibiting SHM is then given by:

$$v = -A\omega \sin(\omega t)$$

The 'minus' sign ensures that 'v' is negative for the first $\frac{1}{2}$ of the cycle.....



...and then positive for the 2nd $\frac{1}{2}$ of the cycle.

Note that setting $t=0$ (at the far edge, where $x=A$), $v = -A\omega \sin(\omega \times 0) = -A\omega \times 0 = 0$, as expected. After $\frac{1}{4}$ of a cycle, the particle will be passing through the equilibrium position (in the negative direction). Setting time, $t = T/4$ gives us this answer from the equation also :

$$v = -A\omega \sin\left[\omega \times \frac{T}{4}\right] = -A\omega \sin\left[\frac{2\pi}{T} \times \frac{T}{4}\right] = -A\omega \sin(\pi/2) = -A\omega$$

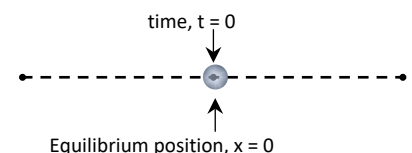
which, of course, is the maximum value for 'v', as expected!

NOTE !!

The ' ω ' in these equations is measured in radians per second, hence, you must have your calculator in 'radians mode' before you use 'sin' or 'cos' functions.

Setting $t = 0$ at other positions

You can choose to 'start the stopwatch' at any value of 'x', e.g. if we set $t=0$, when $x=0$, then we're starting the stopwatch here \rightarrow



The equations then become: $x = A \sin(\omega t)$, and $v = -A\omega \cos(\omega t)$

In general we add a 'phase angle', ϵ , to the angle ' ωt ' in each of the original equations, when the time starts at any other position (in between $x=0$, and the extremities):

$$x = A \cos(\omega t + \epsilon)$$

and

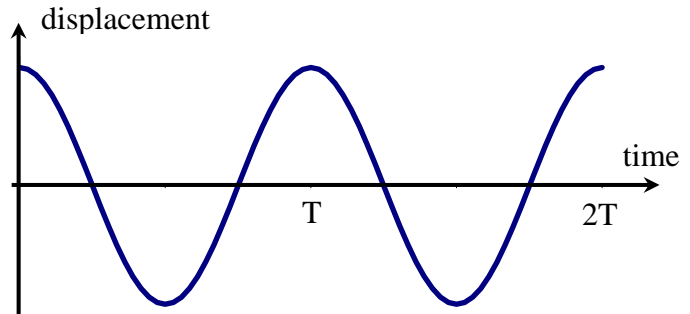
$$v = -A\omega \sin(\omega t + \epsilon)$$

Note : ϵ is an angle and is therefore measured in radians.

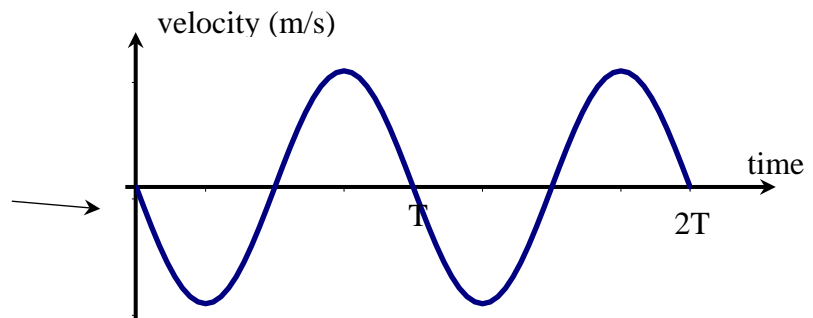
A graphical representation of a, v, and x

The three graphs shown below are always the same **relative to each other**, but the timing can start at any point during an oscillation.

In this case the graphs are for a system, like the simple pendulum, where the timing would normally begin when the displacement is a maximum (when you 'let go' of the pendulum), i.e. when $x = +A$.



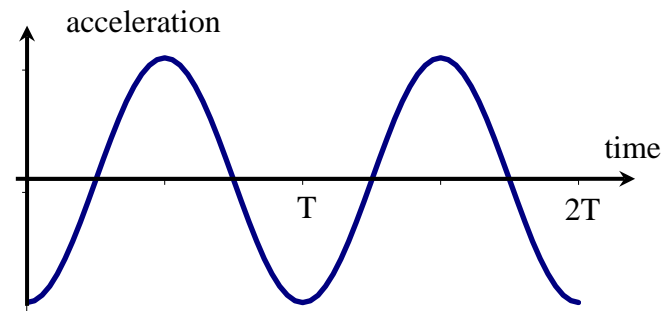
As this is a positive displacement, then the velocity a fraction of a second after letting go would be negative. Hence the velocity graph is zero at the beginning but then gets increasingly negative.



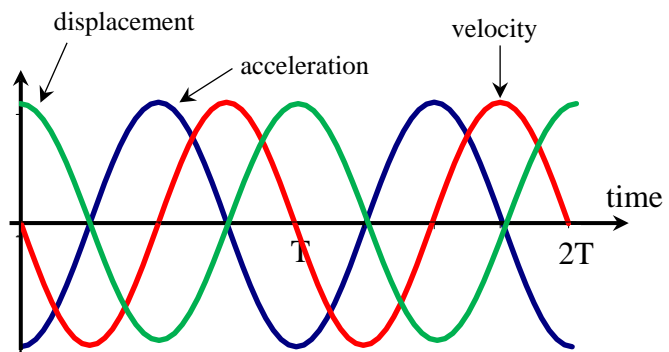
The acceleration-time graph is the inverse of the displacement-time graph since, in SHM,

$$a \propto -x$$

Thus, when x is a maximum, a is a (negative) maximum.



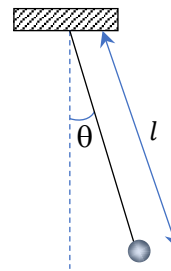
Here are all three graphs superimposed:



2 examples of SHM

A) The simple pendulum

A pendulum length, l , will oscillate approximately with SHM, as long as the release angle, θ , is small.
(The smaller the angle the closer the pendulum approximates SHM).



It can be shown (using the 'small angle approximation') that the acceleration experienced by the pendulum bob, a , is given by: $a = -g x / l$ (g = acceleration due to gravity)

This completely satisfies the conditions for SHM ($a \propto -x$), as g and l are constants in this case. Comparing the equation above to the SHM equation, we can derive (derivation not required) the equation for the period of oscillation of the pendulum:

$$a = -\frac{g}{l} x$$

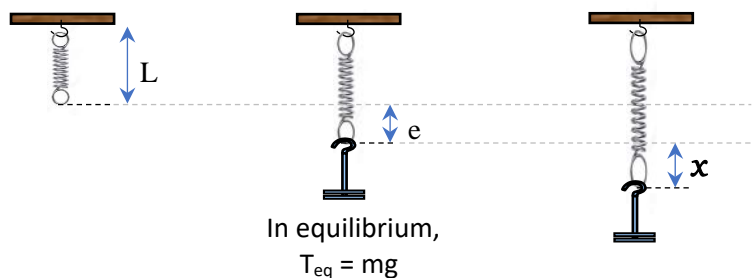
$$a = -\omega^2 x$$

leads to....

$$T = 2\pi \sqrt{\frac{l}{g}}$$

B) Mass-spring system

A metal spring of original length, L , and spring constant, k , is loaded with a mass, m , causing an extension, e . In accordance with Hooke's Law, the tension at equilibrium, $T_{eq} = k e = mg$.



When the mass is pulled down a further distance, x , it is no longer in equilibrium when released, and will oscillate with SHM. The **resultant force** at the point of release is $\Sigma F = T - mg$ (' T ' is the new tension relating to the total extension ' $e+x$ ').

It can be shown that: $a = -k x / m$

This completely satisfies the conditions for SHM ($a \propto -x$), as k and m are constants. Comparing the equation above to the SHM equation, we can derive (derivation not required) the equation for the period of oscillation of the system:

$$a = -\frac{k}{m} x$$

$$a = -\omega^2 x$$

leads to....

$$T = 2\pi \sqrt{\frac{m}{k}}$$

BE AWARE !!

Don't mix 'e' with 'x': $k e = mg$ (at equilibrium) and $k x = ma$ (during oscillations).

Thus, if the mass is given, 'k' cannot be calculated with " $k e = mg$ " unless 'e' is given (rather than 'x').

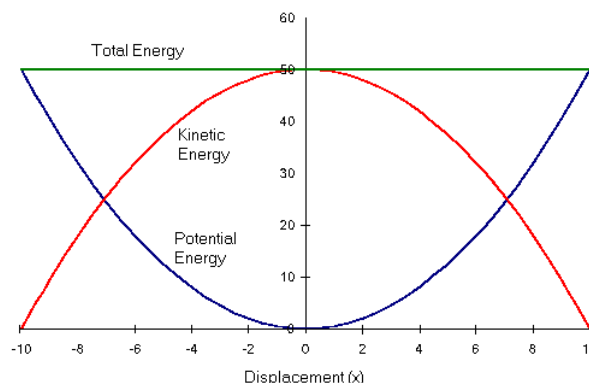
Hence, if **corresponding** values of 'a' and 'x' are given, e.g. from a graph of 'a' against 'x', then the second equation, $k x = ma$, can be used to find 'k'. The only other alternative to finding 'k' is the equation for T above.

Free Oscillations

Free oscillations occur where the total energy of an oscillating system stays constant, but it changes between kinetic energy and potential energy.

The diagram shows how the potential energy and kinetic energy vary with displacement for an object moving with SHM:

Notice that the total energy remains constant as there are no resistive forces acting on the system in this case.

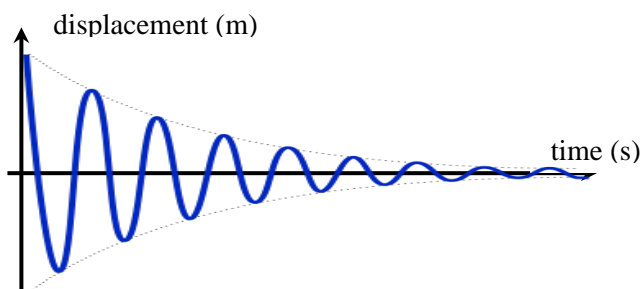


In practice, the amplitude of the oscillations gradually becomes smaller and smaller unless the pendulum is supplied with energy. Air resistance is one reason why the amplitude decreases gradually. For example, for the case of a pendulum, collisions between the bob (&string) and the air molecules transfer kinetic energy from the bob to the air molecules. This gradually reduces the energy of the SHM system, whilst raising the thermal energy of the air particles.

Damping

In most oscillating systems, resistive forces gradually reduce their total energy. This is known as damping, i.e. the system is said to be 'damped'.

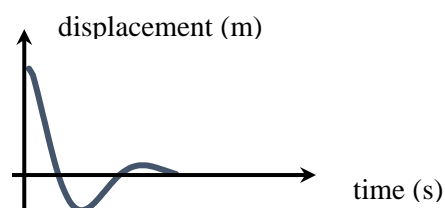
Here's a graph of an oscillating system that is exhibiting '**light damping**' →



Note that the period would be unchanged as compared to the same system with no damping.

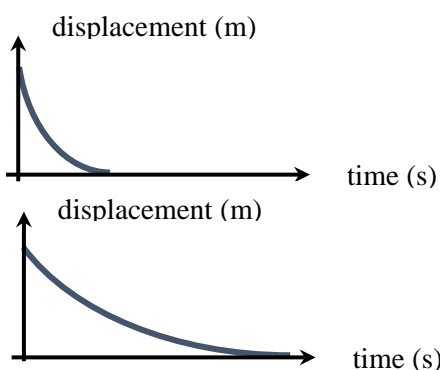
Heavy damping occurs when the resistive forces are significantly greater. The oscillating system is barely able to complete one cycle.

Notice that the period is significantly increased as compared to the lighter damping seen above.



When a system returns to the equilibrium position in the least time possible, where the displacement never becomes negative (no 'overshoot'), the damping is said to be **critical**.

Finally, a greater increase in damping will cause the system to take a long time to return to equilibrium – **overdamping**.



Examples of damping

Light damping

There are many examples, including: a bungee jumper; a child on a swing after the 'pusher' stops pushing; any freely oscillating pendulum; a vibrating string on a musical instrument.



Heavy damping

Swinging doors e.g. in a restaurant, are often designed to 'overshoot' a little before coming to a rest; a (coiled spring) bed mattress allows a little 'oscillation' (as seen when a child uses it as a trampoline!); a 'bouncy castle'.

Critical damping

This is the most precisely-defined type of damping, but there are many examples due to its importance in engineering: car and bike suspension systems; speedometer and rev. counter needles in vehicles; artillery recoil mechanisms; robotics (impact/unexpected motion control); some door closing mechanisms



Overdamping

Earthquake protection in skyscrapers; school/office door closing mechanisms; car boot opening systems

Critical damping

Mountain bike (as well as car) suspension systems are generally designed to be very close to a critical damped system. This is so that if the bike hits a bump or small rock, or if landing after 'catching a little air', then the landing isn't so hard that it hurts the wrists/arms/legs of the cyclist (overdamped), but also isn't too soft so that the bike and biker bounce up and down a few times after impact (light to heavy damping).

The biker thus regains control as quickly as possible.



Resonance

The **natural frequency** of a system is the frequency at which it oscillates without any force applied.

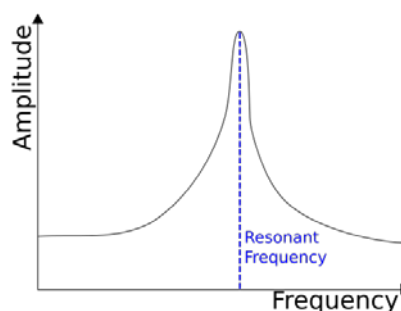
If a system is acted upon by a **sinusoidally varying oscillating force**, it will be forced to vibrate at the **forcing frequency**.

If the forcing frequency is equal to the natural frequency of the system, each push will build up the amplitude further. If there were no damping to dissipate energy, then the amplitude of vibration would increase continuously as the system absorbed energy from the driver. This effect is called resonance.

The graph below shows a typical 'resonance curve' where the oscillating system is forced to oscillate at various frequencies. The 'response' is how large the resulting amplitude is. There's a 'maximum response' when the forced frequency equals the natural frequency of the system.

With no damping, the amplitude and energy of the system will increase continuously.

With damping, the amplitude and energy will increase until energy is being dissipated at the same rate as it is being supplied.

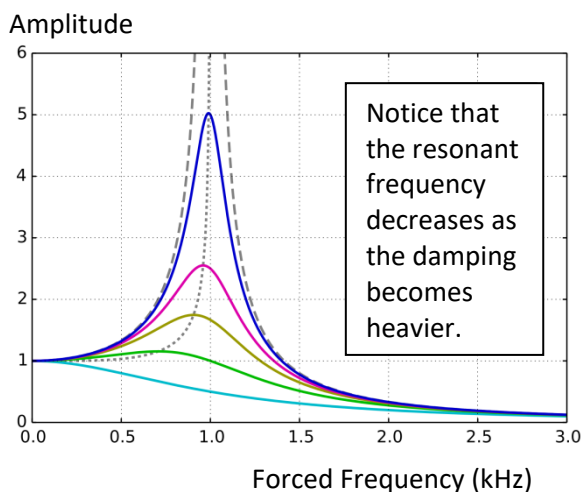


Effect of damping on natural frequency.

Increasing the amount of damping on a system has three effects:

- The amplitude of the peak oscillation decreases.
- The resonance peak gets broader.
- The frequency at which maximum response occurs also decreases.

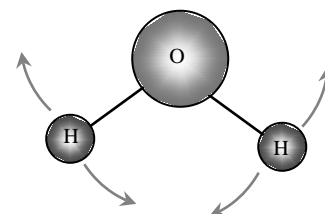
The decrease in amplitude and frequency are more marked with heavier damping.



Examples of resonance

1) A microwave oven

The natural frequency of oscillation of water molecules is around 1×10^{11} Hz. This coincides with the frequency of the microwave region of the EM spectrum. When water molecules are bombarded with microwaves (forcing frequency) the microwaves are readily absorbed, **increasing the amplitude of oscillation, i.e. the water heats up.**



NOTE !!

In reality, the frequency of the microwaves are off-resonance, since if they matched too closely, the heating process is so efficient only the outside layer of the food product would be heated!

2) Circuit tuning (radio receiver)

In an AC circuit, the resistance of the circuit as a whole (impedance) depends on the frequency of the supply voltage. Thus, there's a certain minimum resistance value (maximum current) at a specific frequency. This 'resonant frequency' depends on the values of the capacitor and the inductor in that circuit. When a wide range of radio waves (forcing frequency) are detected by an antenna, only the radio signals very close to the 'natural frequency' of the circuit cause resonance. Thus, different radio station signals can be picked up with each specific value of the capacitor/inductor.

3) The Millennium bridge, London (undesirable resonance)

To celebrate the year 2000 a footbridge was built in London. Unfortunately, the pedestrians walking along the bridge caused the bridge to oscillate (forcing frequency) at the natural frequency of the bridge. This inevitably led to large oscillations (resonance). The bridge was promptly closed, and extra struts were added that significantly changed the natural frequency of the bridge! (A similar, disastrous example is the 1940 Takoma bridge incident, although the mechanism was 'flutter' which is slightly different).



The Millennium bridge, London

Section 3.3 - Kinetic Theory

Overview

The next three sections are a great example of experimental and theoretical physics combining. The first section describes the relationship between the pressure, volume and temperature of a gas (ideal in nature) as observed in real, practical investigations.

The second section describes an equation relating p , V and molecular speed, but is derived from a purely theoretical approach. The two are then combined to show that the temperature of a gas is directly proportional to the average kinetic energy of the particles.

The Equation of State (aka The Ideal Gas Equation)

The following relationships were found by experiments:

	$pV = \text{constant}$	(Boyle/Mariotte's law)	} No knowledge of these individual laws is needed for the exam.
	$\frac{V}{T} = \text{constant}$	(Charles' law)	
and	$\frac{p}{T} = \text{constant}$	(Gay-Lussac's law)	

where p = pressure (pascal, Pa, or N/m^2); V = volume (m^3); T = temperature (kelvin, K)

Therefore the three equations can be summarised by one equation,

$$\frac{pV}{T} = \text{constant}$$

or, alternatively,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

The value of the constant depends on the mass of the gas. Experiments with real gases at low enough pressures show that the constant is the same for all gases if one mole is considered. The constant is called the universal molar gas constant, R , whose value is, $R = 8.31 \text{ J mol}^{-1} \text{ K}^{-1}$. Hence:

$$pV = nRT$$

where n = number of moles

This is known as the 'equation of state', or the 'ideal gas equation'. As the latter name suggests, it only really works for an 'ideal gas', but works very well for many real gases as long as the temperature and pressure aren't too extreme.

NOTE! There are a few 'assumptions' or behaviours exhibited by an ideal gas:

- 1) Collisions are perfectly elastic, so particles bounce off the container and off each other without losing any kinetic energy.
- 2) The gas particles exert forces on each other only when they collide.
- 3) The gas particles are so tiny they take up no space at all (i.e. negligible volume).

Alternative version

Sometimes, the number of particles, N , may be given rather than the number of moles, n . In this case, we use the equation for n :

$$n = \frac{N}{N_A} \quad \text{such that} \quad p V = n R T = \frac{N}{N_A} R T$$

However, R and the Boltzmann's constant, k , are related: $k = R / N_A$, hence:

$$p V = N k T \quad \text{where } k = 1.38 \times 10^{-23} \text{ JK}^{-1}$$

Kinetic Theory

The second equation relating the pressure, p , and volume, V , of a gas, with the average square speed, $\overline{c^2}$, of the gas particles, is derived from a purely theoretical approach. In order to be able to apply fairly simple equations such as Newton's 2nd law ($F = \delta m v / \delta t$), the gas sample being modelled must also be simple, and so many assumptions are made as the foundation on which the relationship is derived. (Some of these are also seen as the assumptions for an ideal gas):

1. The intermolecular collisions are perfectly elastic, as well as collisions with boundaries.
2. Repulsive forces only exist when the particles are close enough, i.e. when they collide.
3. The volume of the molecules themselves can be neglected compared to the volume occupied by the gas.
4. The time taken for a collision is negligible compared with the time spent by a molecule between collisions.
5. A molecule moves with uniform velocity between collisions.
6. There is a large number of molecules even in a small volume, with a large number of collisions in a small time.
7. The motion of the molecules is evenly distributed over all directions.
8. The range of intermolecular forces (both attractive and repulsive) is small compared to the average distance between molecules.
9. **There is a random distribution of energy among the particles.**

Also assumptions for an ideal gas!

The equation derived from this 'model' is then:

$$p = \frac{1}{3} \rho \overline{c^2}$$

where p = pressure (N/m^2), $\overline{c^2}$ = mean square speed (m^2s^{-2}), and

$$\rho = \text{density (kgm}^{-3}\text{)} = \frac{\text{total mass of gas}}{\text{total volume of gas}} = \frac{M_T}{V} = \frac{N m}{V}$$

where N = number of particles in the gas, m = mass of **each individual particle**

Combining the two equations (experiment & theory)

First, the equation for density, ρ , seen at the bottom of the previous page, is needed:

$$\rho = \frac{M_T}{V} = \frac{N m}{V} \quad \therefore p = \frac{1}{3} \rho \overline{c^2} = \frac{1}{3} \frac{N m}{V} \overline{c^2}$$

$$\therefore p V = \frac{1}{3} N m \overline{c^2} \quad \text{----- Equ. [1]}$$

Here's the ideal gas equation: $p V = n R T$ ----- Equ. [2]

Since the left hand side of equations [1] and [2] are both ' pV ', we can equate the right hand sides:

$$n R T = \frac{1}{3} N m \overline{c^2}$$

Multiplying by 3/2: $\frac{3}{2} n R T = \frac{1}{2} N m \overline{c^2}$

Re-arranging the RHS: $\frac{3}{2} n R T = N \times \frac{1}{2} m \overline{c^2}$

Since N = number of particles in the gas, and " $\frac{1}{2} m \overline{c^2}$ " = the mean kinetic energy of each particle, the RHS ($N \times \frac{1}{2} m \overline{c^2}$) must equal the **total kinetic energy of all the gas particles**.

Hence, for a monatomic gas, the total translational KE = $\frac{3}{2} n R T$. Later, we will see that this is known as the 'internal energy' (for a monatomic gas), U . Hence:

$$U = \frac{3}{2} n R T$$

or for 1 mole ($n=1$):

$$U = \frac{3}{2} R T$$

We can re-arrange a little further:

$$\frac{3}{2} n R T = N \times \frac{1}{2} m \overline{c^2}$$

Dividing by N : $\frac{3}{2} \frac{n R T}{N} = \frac{1}{2} m \overline{c^2}$ and since $n = N / N_A$

$$\frac{3}{2} \frac{R T}{N_A} = \frac{1}{2} m \overline{c^2} \quad \text{and since } k = R / N_A$$

$$\frac{3}{2} k T = \frac{1}{2} m \overline{c^2}$$

In other words, the average KE of **each particle** = $\frac{3}{2} k T$, where k = the Boltzmann's constant.

Examples

1. A gas cylinder contains $2.5 \times 10^{-4} \text{ m}^3$ of Helium at a pressure of 20 atmospheres (1 atmosphere = 101 000 Pa). If there are 0.2 moles of helium in the cylinder, calculate the temperature of the gas.

Re-arranging $pV = nRT$, gives

$$T = pV / nR = (202000)(2.5 \times 10^{-4}) / (0.2 \times 8.31) = 309 \text{ K}$$

2. The table shows the distribution of molecular speeds among 15 molecules of an ideal gas.

Number of molecules	2	4	5	3	1
Speed (ms^{-1})	200	300	500	600	700

- a) Calculate the mean square speed $\overline{c^2}$.
 b) Calculate the pressure exerted by the gas if $\overline{c^2}$ for all its molecules is the same as that calculated in part (a), and the density of the gas is 1.25 kgm^{-3} .

$$\text{a) } \overline{c^2} = \frac{(2 \times 200^2) + (4 \times 300^2) + (5 \times 500^2) + (3 \times 600^2) + 700^2}{15} = \frac{3\,260\,000}{15} = 217\,333 \text{ m}^2\text{s}^{-2}$$

$$\text{b) } p = \frac{1}{3} \rho \overline{c^2} = \frac{1}{3} \times 1.25 \times 217333 = 90\,600 \text{ Pa (3 s.f.)}$$

3. A vessel of volume $1.0 \times 10^{-3} \text{ m}^3$ contains helium gas at a pressure of $2.0 \times 10^5 \text{ Pa}$ when the temperature is 300K. (Relative atomic mass of helium = 4)

- a) Calculate the number of helium atoms in the vessel.
 b) Calculate the mass of helium in the vessel, given that $1u = 1.66 \times 10^{-27} \text{ kg}$.
 c) Calculate the r.m.s. speed of the helium atoms.

$$\text{a) } n = pV / RT = 0.080 \text{ mol} \quad \therefore n = N / N_A \text{ gives } N = n \times N_A = 0.080 \times 6.02 \times 10^{23} = 4.8 \times 10^{22}$$

$$\text{b) (approximately) total mass, } M_T = N \times m = N \times (4u) = 4.8 \times 10^{22} \times 6.64 \times 10^{-27} = 3.2 \times 10^{-4} \text{ kg}$$

$$\text{c) } p = \frac{1}{3} \rho \overline{c^2} = \frac{1}{3} \frac{M_T}{V} \overline{c^2} \quad \text{Hence, } \overline{c^2} = \frac{3pV}{M_T} = \frac{3 \times (2 \times 10^5) \times (1 \times 10^{-3})}{3.2 \times 10^{-4}} = 1\,875\,000 \text{ m}^2\text{s}^{-2}$$

However, the question asks for "ROOT mean square speed" (r.m.s.), hence:

$$c_{\text{rms}} = \sqrt{\overline{c^2}} = 1370 \text{ ms}^{-1}$$

NOTE!

- 1) Don't mix 'M' with 'm' (e.g. in questions like 3(b)).

M_T = total mass of gas; m = mass of an individual particle in the gas.

(Also, as in chemistry, **M** = molar mass).

- 2) You can also find ' M_T ' in 3(b) by using the number of moles, and the molar mass (which you get from "relative atomic mass = 4") – see next page!

Other definitions and equations needed

The mole is defined thus:

The mole is the S.I. unit of an 'amount of substance'. It is the amount containing as many particles (e.g. molecules) as there are atoms in 12 g of carbon-12.

Avogadro's constant is defined thus:

This is the number of particles per mole. ($N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$)

Relative atomic mass

The relative molecular/atomic mass, M_R , is a number that states precisely how a molecule or atom's mass compares to 1/12th the mass of a carbon-12 atom. Since it's 'relative', it has no units.

For example, for helium, $M_R(\text{He}) = 4.0026$. However, this is often rounded to just '4'.

This means that one atom of helium has a mass that is exactly 4.0026 times larger than 1/12th the mass of a carbon-12, i.e. approximately, it's about 4 times more mass. This can be useful to find the mass of one mole of a substance, M , since the mole is defined in such a way so that,

$$M = M_R / 1000$$

(M has units of kgmol^{-1})

Example

1) The relative **molecular** mass for oxygen, $M_R(\text{O}_2) = 31.9988$. Calculate the molar mass of O_2 .

$$M = M_R / 1000 = 31.9988 / 1000 = 0.0319988 \text{ kgmol}^{-1}$$

2) Taking the relative **atomic** mass of nitrogen as $M_R(\text{N}) = 14$, calculate the mass of one mole of atomic nitrogen. (Note: the '14' is a rounded value).

$$M = M_R / 1000 = 14 / 1000 = 0.014 \text{ kgmol}^{-1}$$

Alternatively: If $M_R = 14$, this is a rounded, hence approximate, value. So, one atom of nitrogen has a mass of (approx.) 14u. Since there are N_A atoms in a mole, the total mass of one mole of nitrogen (atomic) is approximately,

$$M = N_A \times 14 \text{ u} = (6.02 \times 10^{23}) \times 14 \times (1.66 \times 10^{-27}) = 0.01399 = 0.014 \text{ kgmol}^{-1} \quad (\text{to 2 s.f.})$$

Also, remember that:

$$n = \frac{N}{N_A}$$

$$n = \frac{M_T}{M}$$

where M_T = total mass of gas, M = molar mass

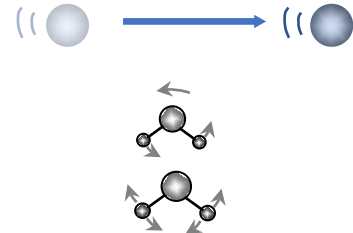
Section 3.4 - Thermal physics

Internal energy

The internal energy of a system is the energy of the molecules or atoms. It has two components:

(a) **Kinetic energy** due to;

- i. Translational motion of its molecules.
- ii. Rotational motion of its molecules.
- iii. Vibrational motion of its molecules.



All these forms of K.E. depend on the temperature. In a monatomic gas, e.g. Neon or Argon, only translational K.E. is present.

(b) **Potential energy** due to intermolecular forces.

In an **ideal gas** there are no forces between the molecules therefore there is no potential energy. Therefore the internal energy, U , of an ideal gas is wholly kinetic. Furthermore, if the gas is also monatomic (like Neon or Argon), then the KE is just translational, hence,

$$U = \frac{3}{2} n R T$$

(as seen in section 3.2)

Absolute zero is the temperature of the system when the internal energy is at a **minimum**.

Note that although this coincides with zero KE, the PE may be negative – hence the term ‘minimum’ rather than ‘zero’.

Flow of energy

Energy can be transferred to or from a system as 1) heat or 2) work, and afterwards it is impossible to tell which form it took. For example, air in a bicycle pump can be heated either by compressing the piston or by transferring heat energy by conduction, convection or radiation.

1. Flow of Heat

Heat energy, Q , is the energy which flows by conduction, convection or radiation from one body to another because of the temperature difference between them. **It is energy in transit, and not contained within the system.**

Heat will always flow from a body at high temperature to a body at a lower temperature. It will continue to flow until the two bodies are in **thermal equilibrium**.

At thermal equilibrium, the systems in contact will be at the same temperature, therefore no heat will flow.

2. Work

Work is the energy that is transferred from one system to another by a force moving its point of application in its own direction.

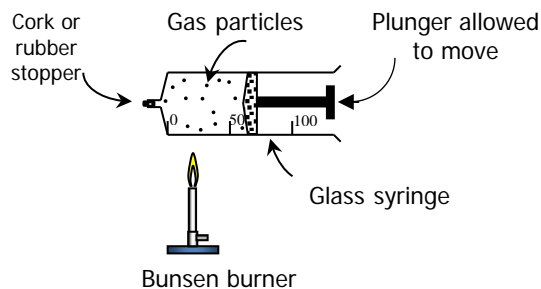
Calculating work done

It can be shown that if heat is introduced to a gas where the set-up is such that the **pressure, p , remains constant** (an **isobaric** change), the gas will do work (by expanding) in accordance with the following equation:

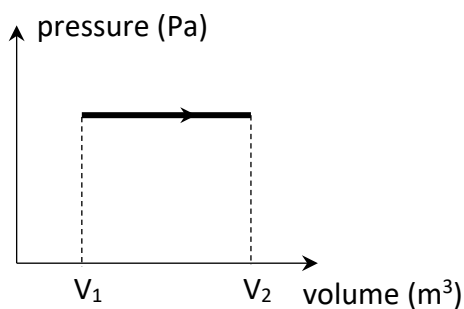
$$W = p \Delta V$$

where ΔV = change in volume (m^3)

In practise, the situation above is quite realistic and easy to set up. Place a good quality, glass syringe (very low friction) in a source of heat e.g. a Bunsen flame, and the heat that flows into the gas causes it to expand.



It is often convenient to draw a graph of pressure, p , versus volume, V . These graphs are known as **indicator diagrams**. Here's the indicator diagram for the situation above:



$$W = p \Delta V = p (V_{\text{final}} - V_{\text{initial}}) = p (V_2 - V_1)$$

Notice that the work done is equal to the **area under the graph**. This is true even if the pressure is not constant. It is important, therefore, to remember this fact, as it is often needed in the exam.:

The work done is always equal to the area under the p - V graph

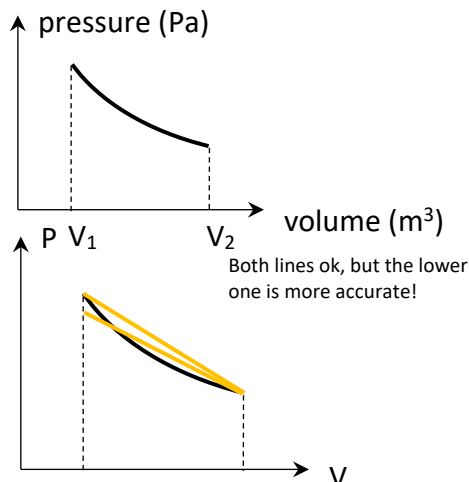
Calculating the area under curved sections

There are often curved sections to an indicator diagram. For example, a change in pressure and volume that occurs at constant temperature (an **isothermal** change) looks like this →.

If you're asked to calculate the work done in this type of graph, the best you can do is an approximation. The area under the curved line is approximately equal to the area under one of the straight lines shown.

The area below the graph is a trapezium, hence,

$$\text{Work} = \text{Area} = \frac{(p_1 + p_2)}{2} \times \Delta V$$



The 1st law of thermodynamics

This is essentially the conservation of energy applied to a contained gas system.

Heat given to a system = Increase in internal energy of the system + Work done by the system

In equation form:

$$Q = \Delta U + W$$

or re-arranged...

$$\Delta U = Q - W$$

where,

Q = heat energy given **to** the system; ΔU = **change** in internal energy of the system

W = work done **by** the system

Positive or Negative?

Q is + **ve** if heat is supplied to the gas, - **ve** if heat is transferred from it.

W is + **ve** if external work is done by the gas (expanding); - **ve** if work is done on it (compressing).

ΔU is + **ve** if the internal energy increases, and - **ve** if the internal energy decreases.

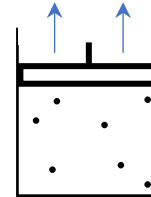
Examples (including changes of state that are : isobaric, isothermal, isovolumetric, and adiabatic)

A) Constant pressure (isobaric)

Since the pressure doesn't change, the work done is easy to calculate with $W = p \Delta V$. Applying the 1st law:

$$Q = \Delta U + W \quad \rightarrow \quad Q = \Delta U + p\Delta V$$

Frictionless plunger/piston allowed to move freely



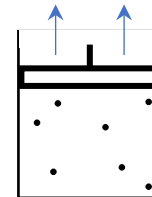
Heat

B) Constant temperature (isothermal)

If, for example, the external pressure decreased, the gas will expand. If this is done **very slowly**, any slight decrease in the gas' temperature will cause heat to flow in to maintain thermal equilibrium, hence, $\Delta U = 0$. Applying the 1st law:

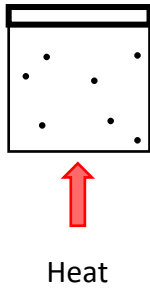
$$Q = \Delta U + W \quad \rightarrow \quad Q = W$$

Frictionless plunger/piston allowed to move freely



Meaning, that all the heat entering the gas is used to do work **by the gas** as the piston moves outward.

C) Constant volume (isovolumetric)



Since the volume doesn't change, no work can be done by the gas, hence, $W = 0$. Applying the 1st law:

$$Q = \Delta U + W \quad \rightarrow \quad \boxed{Q = \Delta U}$$

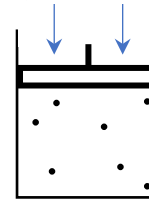
Meaning, that all the heat entering the gas converts to an increase in the internal energy.

D) No heat enters or leaves the system (adiabatic)

This is achieved by pushing in or pulling out the piston **very quickly**. This means that heat doesn't have enough time to move in or out of the system, hence, $Q = 0$.

In the case of pushing the piston in quickly, work is done **on** the gas by an external force, and so, the work, W is **negative**. Applying the 1st law:

Frictionless plunger/piston pushed in **quickly** by an external force



$$Q = \Delta U + W \quad \rightarrow \quad \boxed{\Delta U = -W}$$

This means, that as 'W' has a negative value, there's a double negative here, making the ΔU value positive, which is what we would expect – suddenly pushing in the piston on a bicycle pump would make the air inside the pump hotter (not colder)!

NOTE !!

For liquids and solids, there's very little change in volume, and so the work done is zero (or very close to it), therefore, applying the 1st law: $Q = \Delta U$

Specific heat capacity

This is a quantity that tells us the amount of heat needed to produce a temperature rise of one degree in one kilogram of a substance. It's described in the WJEC definitions document as follows:

The heat required, per kilogram, per degree celsius or kelvin, to raise the temperature of a substance. Unit = $\text{J kg}^{-1} \text{K}^{-1}$.

Hence, the equation is:

$$Q = m c \Delta\theta$$

where Q = heat input (J); m =mass (kg);
 c = specific heat capacity ($\text{J kg}^{-1} \text{K}^{-1}$);
 $\Delta\theta$ = change in temperature (K or $^{\circ}\text{C}$)

Note that water has one of the highest values for specific heat capacity ($c_{\text{water}} = 4200 \text{ J kg}^{-1} \text{K}^{-1}$).

Example

Calculate the heat required to raise 120kg of water from 20°C to 50°C .

$$Q = m c \Delta\theta = 120 \times 4200 \times 30 = 1.512 \times 10^7 \text{ J}$$

Section 3.5 - Nuclear decay

Notations

The A_ZX notation is used to represent elements where,

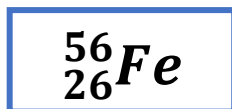
- A = Nucleon number (total number of nucleons = protons + neutrons; aka mass number)
- Z = Proton number (aka, atomic number)
- X = Chemical symbol of the element

It is important to remember that every element has a unique proton number (Z) which will not change. If the number of protons change, the element will change with it.

However, the nucleon number (A) can change as the number of neutrons (N) can change.

An **isotope** is an element with the same number of protons, but a different number of neutrons.

For example,



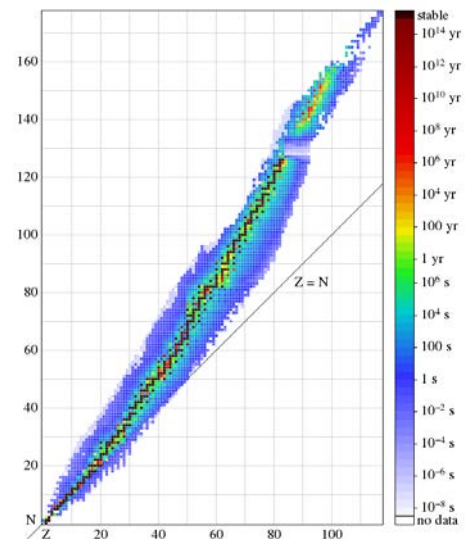
So, nucleon number = 56
 atomic number = 26 (number of protons)
 "Fe" = chemical symbol for 'Iron'
 \therefore number of neutrons = 56-26 = 30

Radioactive Decay

Within the nucleus there are positively charged protons, which repel each other due to electrostatic repulsion. But as the nucleus does not blow itself apart, there must be another (attractive) force between the nucleons - this is the strong nuclear force.

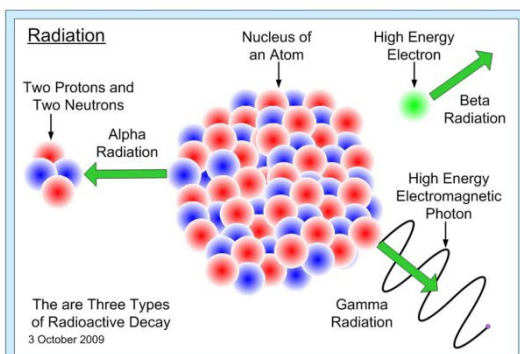
These two forces are balanced in a stable nucleus, but an imbalance makes the nucleus unstable. There's a certain value for the neutron:proton ratio that makes the two forces in a nucleus balanced, and this ratio increases with heavier nuclei, as shown by the graph opposite \rightarrow .

Radioactive decay is the process by which an unstable nucleus becomes more stable by



spontaneously decaying into a daughter nucleus while emitting particles such as alpha or beta, and/or energy, e.g. gamma rays.

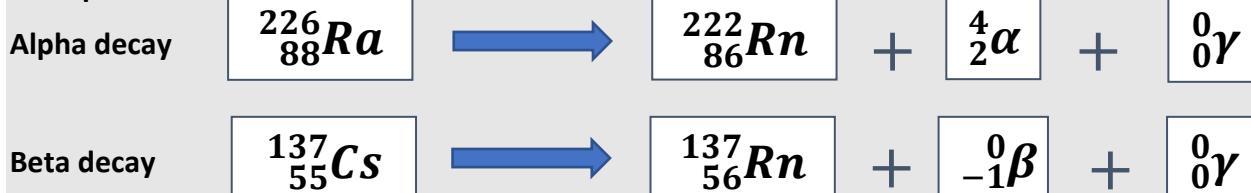
Radioactive decay usually produces alpha and gamma, or beta and gamma. Sometimes just a gamma ray is emitted. (All three are shown together in the diagram on the left).



Properties of Alpha, Beta and Gamma radiation

	alpha	beta	gamma
Symbol	α	β	γ
What is it?	Helium nucleus	Fast electron	EM wave
Mass (u)	~ 4	~ 0.00055	0
Charge (e)	2+	1 -	None
Speed (% of c)	~ 10%	50-95%	100%
Penetration	Low	Medium	High
Change in nucleon number (ΔA)	- 4	0	0
Change in proton number (ΔZ)	- 2	+1	0
Affected by electric/magnetic fields?	Yes, both	Yes, both	Neither

Examples



Penetration vs Ionisation

If a particular radiation is highly ionising, it interacts strongly with any matter it passes through. This means that some of the KE of the radiation particle is absorbed with each interaction, which in turn means that the radiation has low penetrating power.

α particles have very low penetration (or are highly ionising) compared to β particles for 3 reasons:

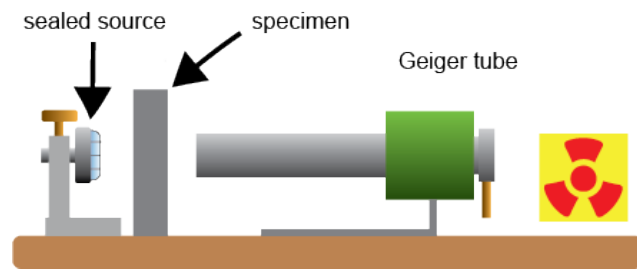
- 1) They have a higher charge than β particles (γ has no charge), which means they interact strongly with other charged particles, e.g. ions or electrons in orbit around atoms.
- 2) They have a larger mass, which means they are more likely to collide with other particles.
- 3) They move slower, again increasing the likelihood of interacting with other particles.

γ rays are weakly ionising (hence have high penetrating power) since they are uncharged, and have no rest mass. Their frequency is very high and so electrons (e.g. in orbits around atoms) can't move quickly enough to interact with them, so they are unlikely to be absorbed or scattered.

Distinguishing between α , β , and γ

Method 1: Penetrating power

The three radiation types have very different penetrating powers. This can be used to determine whether a source is giving out α , β , or γ , or a combination of them. Shown opposite is a typical set-up, using different absorbers in turn.



The background radiation should be measured first, and then deducted from each reading. The count rate should be measured first when there's no absorber in place.

If the radiation count decreases at all after inserting paper or card, then α is present.

If the radiation count decreases further after inserting a sheet of aluminium, then β is present.

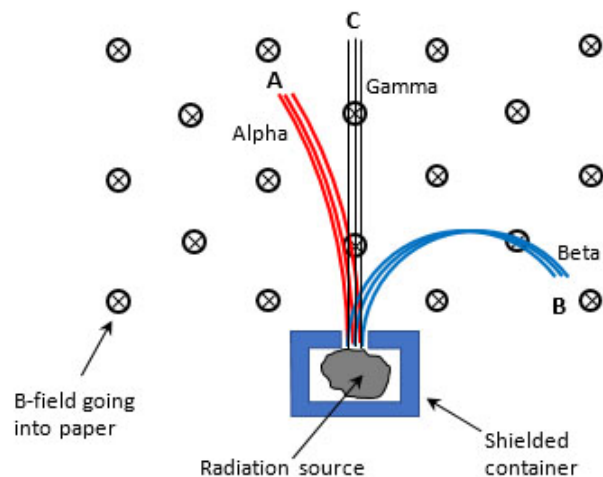
What's left (above background) must be gamma, however as an extra measure, a sheet of lead can be inserted which should further decrease the count rate, confirming the presence of γ .

NOTE !! Not all γ radiation is absorbed, even with a few centimetres of lead.

Method 2: Magnetic/Electric fields

The apparatus opposite may be used to demonstrate the deflection of radiation by a magnetic field.

Without a magnet the count produced after, say, one minute is recorded (position C). The magnet is then placed in position and the count rate at C reduces, but will rise again when the detector is moved to a new position such as A.



If the count rate increases somewhere around position A, then the source is emitting α -particles.

If the count rate increases somewhere around position B, then the source is emitting β -particles. γ -rays would not be deflected since they have no charge, and so would be detected at C.

Note that detecting α -particles is more difficult because the deflection is a lot smaller and they are easily absorbed by even 10 cm of air. The deflection is smaller since α -particles have a much greater mass than the β -particles (albeit twice the charge).

After removing the magnet, an electric field could be set up across the area above the source. This wouldn't affect the gamma rays, but the alpha particles would be deflected in the direction of the E-field, whereas the beta particles would be deflected the opposite way. This is an alternative way to the magnetic field method above, but produces similar results.

Radioactive decay

Half-life, $T_{1/2}$

The half-life of a radioactive sample is defined as either:

The time taken for half the number of radioactive nuclei present to decay.

or

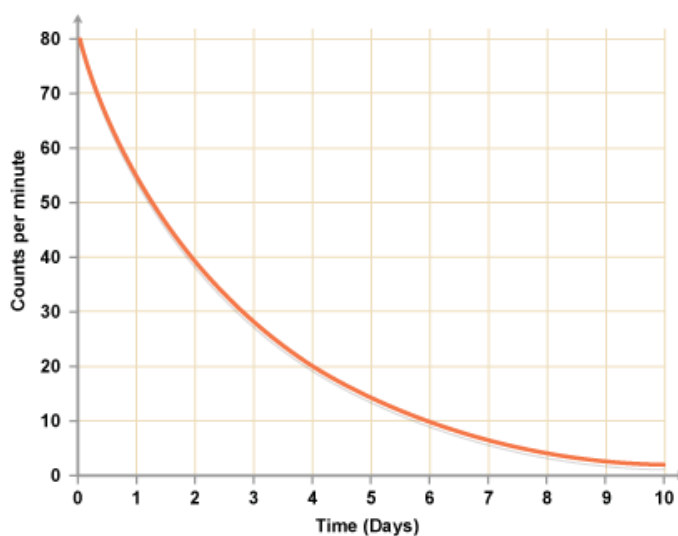
The time taken for the activity to decrease to half of its initial value.

Half-life values have a very wide range, from nanoseconds to millions of years. The half-life is unique to each isotope, and a measurement of this value is useful in identifying specific radioactive isotopes.

The idea of half-life is illustrated in the graph opposite.

The half-life here is 2 days, so, it takes 2 days for the count rate to decrease from 80 to 40 cpm, and from 40 to 20 cpm, etc.

In fact, this works from any starting point, e.g. the graph passes '50cpm' at about 1.3 days; it passes '25cpm' at about 3.3 days, i.e. 2 days later.



Decay equations

The graph above shows that, although radioactive decay is both **spontaneous** (occurs without any impetus) and **random** (no way of knowing **when** the next decay will occur or **which** nucleus will decay next), since there is usually a very large number of unstable nuclei involved, the rate of decay is very predictable.

The rate of decay (number of disintegrations per second) of a sample of radioactive nuclei is known as the ACTIVITY, A. Unit: becquerel (Bq) = s^{-1}

The rate of decay (activity) of a given nuclide at any time is directly proportional to the number (N) of unstable nuclei of that nuclide present at that time. This can be expressed as an equation:

$$A = -\lambda N$$

where

N = number of unstable nuclei present
 λ = the decay constant; Unit = s^{-1}

The negative sign ensures that N decreases as time increases.

Note also that by definition, **activity, A = dN/dt** .

Hence,

$$\frac{dN}{dt} = -\lambda N$$

This type of relationship always leads to exponential decay. There are many examples of this kind of relationship (exponential decay) in real life, e.g. discharging a capacitor; damped oscillations; (enzyme-catalysed) chemical reactions; temperature change in an object hotter than its surroundings; rate of decrease of a beer 'froth'!?

The above equation can be solved by integration, giving the exponential form:

$$N = N_0 e^{-\lambda t}$$

where N_0 = number of unstable nuclei at time, $t = 0$

A simple substitution using $A = -\lambda N$, and $A_0 = -\lambda N_0$, yields an alternative version in terms of activity:

$$A = A_0 e^{-\lambda t}$$

where A_0 = activity (Bq) at time, $t = 0$

If the number of half-lives that have passed, x , (not necessarily an integer) is known, then a shortcut to finding the number of unstable nuclei or activity after a time, t , is:

$$N = \frac{N_0}{2^x}$$

or

$$A = \frac{A_0}{2^x}$$

Calculating half-life, $T_{1/2}$

Using the equation $N = N_0 e^{-\lambda t}$ we can derive an equation which relates the half-life to the decay constant. (This derivation is a requirement of the specification).

After a time equal to one half life, $N = N_0 / 2$, and $t = T_{1/2}$, and so:

$$\frac{N_0}{2} = N_0 e^{-\lambda T_{1/2}} \quad \therefore \quad \frac{1}{2} = e^{-\lambda T_{1/2}}$$

\therefore Taking logs: $\ln(1/2) = -\lambda T_{1/2}$... taking the minus to the right hand side...

$$\ln(2/1) = \lambda T_{1/2}$$

\therefore

$$T_{1/2} = \frac{\ln(2)}{\lambda}$$

Examples

1. The half-life of iron-55 is 2.6 years. Calculate,

(a) The radioactive decay constant (λ).

(b) The number of atoms in a sample of 1.0 g of iron-55 ($N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$).

(c) The initial number of decays per second for 1 g of iron-55.

(a) $\lambda = \ln(2) / T_{1/2} = \ln(2) / (2.6 \times 365 \times 24 \times 3600) = 8.45 \times 10^{-9} \text{ s}^{-1}$

(b) The molar mass, M , of iron-55 must be (approx.) $55 \text{ g} = 0.055 \text{ kg}$.

Number of moles, $n = M_T / M = 0.001 / 0.055 = 0.0182 \text{ mol}$.

$\therefore N = n \times N_A = 0.0182 \times (6.02 \times 10^{23}) = 1.1 \times 10^{22}$

(c) $A_0 = -\lambda N_0 = (8.45 \times 10^{-9}) \times (1.1 \times 10^{22}) = 9.3 \times 10^{13} \text{ Bq}$

2. (Taken from the PH5, June 2016 paper).

Polonium-211 decays to lead-207 with a decay constant (λ) of 1.343 s^{-1} .

(a) Calculate the half-life of polonium-211. [2]

(b) Calculate the initial activity of $4.22 \times 10^{-11} \text{ kg}$ of polonium-211. [3]

(c) Calculate the percentage of polonium-211 nuclei remaining after 2.4 s. [2]

(d) Calculate the time taken for the number of polonium nuclei to decrease to 0.1% of their initial number. [2]

(e) Explain why $4.22 \times 10^{-11} \text{ kg}$ of polonium-211 could be highly dangerous even though it emits alpha particles which cannot penetrate human skin. [2]

(a) $T_{1/2} = \ln(2) / \lambda = \ln(2) / 1.343 = 0.516 \text{ s}$

(b) $n = M_T / M = 4.22 \times 10^{-11} / 0.211 = 2.00 \times 10^{-10} \text{ mol}$

$\therefore N = n N_A = (2.00 \times 10^{-10}) \times (6.02 \times 10^{23}) = 1.20 \times 10^{14}$

$\therefore A_0 = -\lambda N_0 = 1.343 \times (1.2 \times 10^{14}) = 1.62 \times 10^{14} \text{ Bq}$

(c) $N/N_0 = e^{-\lambda t} = e^{-(1.343 \times 2.4)} = 0.0398 \quad \therefore \% \text{ left} = 3.98\%$

(d) $0.1\% = 0.001$, hence $N/N_0 = 0.001$. $\therefore \ln(N/N_0) = -\lambda t \quad \therefore \ln(N_0/N) = +\lambda t$

$\therefore t = \ln(1/0.001) / 1.343 = 5.14 \text{ s}$

(e) If inhaled/ingested, then the large activity, and the fact that alpha radiation is highly ionising, would cause a lot of damage to the cells inside the body.

Section 3.6 - Nuclear Energy

The equivalence of energy and mass

In 1905 Einstein made the startling suggestion that energy and mass are equivalent. He predicted that if the energy of a body changes by an amount E , its mass changes by an amount m given by the equation,

$$E = m c^2$$

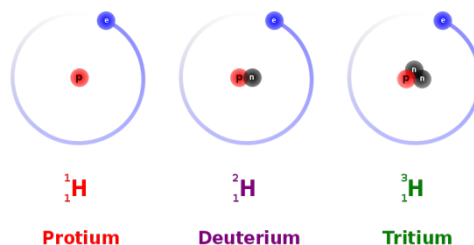
where c = the speed of light ($3.00 \times 10^8 \text{ ms}^{-1}$)



This doesn't just mean that the energy produced in a nuclear reaction (usually as the KE of the products) comes at the expense of a little loss of mass, but that it also works in reverse. If we collide particles in a particle accelerator, some of the KE of the incident particles can be converted into mass, whereby new particles are created at the moment of collision!

Binding Energy

${}^1_1\text{H}$ is the only nucleus which contains a single nucleon. All the other elements, including the other two Hydrogen isotopes, have more than one nucleon in the nucleus, held together by the strong nuclear force. This means that a significant amount of energy would have to be inputted to separate the nucleons in a nucleus so that they were all 'free' (unbound). The energy needed to completely separate the nucleons in a nucleus is known as the **binding energy (BE)**.



The consequence of inputting energy to separate the nucleons is that their mass increases a little once they're separated (since energy and mass are equivalent). Hence, nuclei have a little less mass than the sum of the mass of their constituent (separated) parts. This 'mass loss', correctly known as **mass defect**, can therefore be used to calculate the binding energy (BE).

Example

The masses of a proton, neutron, and a Helium **nucleus** are, $m_p = 1.6726 \times 10^{-27} \text{ kg}$, $m_n = 1.6749 \times 10^{-27} \text{ kg}$, and $m_{\text{He}} = 6.6465 \times 10^{-27} \text{ kg}$ respectively. Calculate

- (i) the mass defect, MD, (ii) the binding energy, BE.

Mass/energy needed to be 'added' to separate the nucleus.

- (i) mass of Helium nucleus + MD = mass of 2 protons + mass of 2 neutrons
 $\therefore \text{MD} = (2m_p + 2m_n) - m_{\text{He}} = 2(1.6726 \times 10^{-27}) + 2(1.6749 \times 10^{-27}) - 6.6465 \times 10^{-27}$
 $\therefore \text{MD} = 4.85 \times 10^{-29} \text{ kg}$
- (ii) Since $E = m c^2$, $\text{BE} = \text{MD} \times c^2 = (4.85 \times 10^{-29}) \times (9 \times 10^{16}) = 4.365 \times 10^{-12} \text{ J}$

The unified atomic mass unit (u)

In nuclear physics mass is measured in a different unit, called the unified atomic mass unit (u). It is based on a scale which ranks the atoms according to their masses, using the common isotope carbon-12 as a reference. On this scale, carbon-12 is assigned a mass of exactly 12 u, and all other masses are expressed relative to this.

Therefore, **1 u** is defined as **one twelfth** ($1/12^{\text{th}}$) of the mass of a carbon-12 atom. Hence,

$$1 \text{ u} = 1.66 \times 10^{-27} \text{ kg}$$

$E = mc^2$ can now be used to find how much energy '1u' is equivalent to:

$$E = m c^2 = (1.66 \times 10^{-27}) \times (3.00 \times 10^8)^2 = 1.494 \times 10^{-10} \text{ J}$$

converting this into electron-volts $\rightarrow E = 1.494 \times 10^{-10} / 1.60 \times 10^{-19} = 931\,000\,000 \text{ eV} = \mathbf{931 \text{ MeV}}$

\therefore

$$1 \text{ u} = 931 \text{ MeV}$$

Note!!

If you check, this answer actually comes out as 934 MeV ! However, if more accurate values for the speed of light, the mass of '1u' and 'e' are used, it works out as 931 MeV!

This is a very useful conversion since atomic masses are often given in unified atomic mass units. If you are given atomic masses in terms of "u" then you do not need to use $E = mc^2$, since it's already been used, as shown above!

Simply multiply the mass defect (in 'u's) by 931 to find the binding energy in MeV.

Example

Using the mass values below, calculate the binding energy of an iron (${}_{26}^{56}\text{Fe}$) nucleus:

$$m_{\text{proton}} = 1.00728\text{u}, \quad m_{\text{neutron}} = 1.00866\text{u}, \quad m_{\text{Fe}} = 55.92067\text{u}.$$

$$\mathbf{MD = (26 \times m_{\text{proton}} + 30 \times m_{\text{neutron}}) - m_{\text{Fe}} = 26.18928 + 30.2598 - 55.92067 = 0.52841\text{u}}$$

$$\therefore \text{Binding energy, BE} = 931 \times \text{MD} = 931 \times 0.52841 = 492 \text{ MeV}$$

In summary

If masses are given in,

kg \rightarrow Calculate the **mass defect**, then use **$\text{BE} = \text{MD} \times c^2$** **Answer in Joules**

'u's, \rightarrow Calculate the **mass defect**, then use **$\text{BE} = 931 \times \text{MD}$** **Answer in MeV**

Binding energy per nucleon

The total binding energy is linked to the size of the nucleus. The more nucleons there are, the greater the energy needed to separate them all out.

A more useful comparison is the **binding energy per nucleon**. This is the average energy needed to remove each nucleon from the nucleus. Hence, the higher the binding energy per nucleon, the more stable the nucleus, as each nucleon is more strongly bound to its neighbours. It is easy to calculate once you've calculated the binding energy for a particular nucleus:

$$\text{BE per nucleon} = \frac{\text{BE}}{\text{number of nucleons (A)}}$$

Example

On the previous page, the binding energy, BE, for iron-56 was calculated as 492 MeV. What's the binding energy per nucleon for this nucleus?

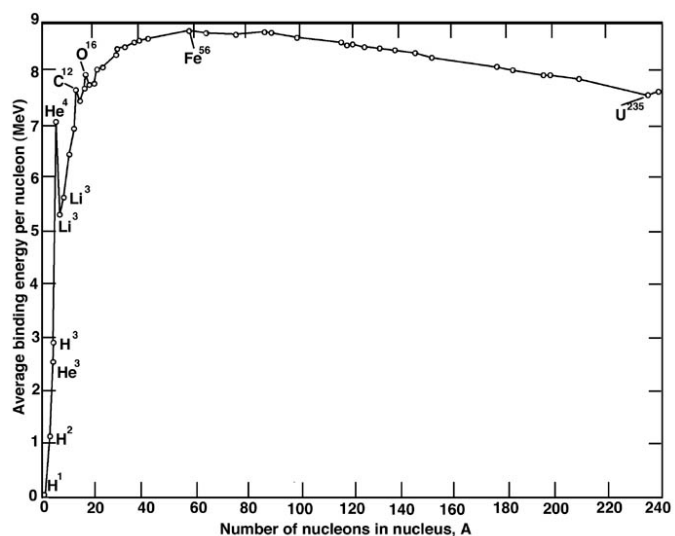
$$\text{BE per nucleon} = \text{BE} / \text{A} = 492 / 56 = 8.79 \text{ MeV}$$

If a graph is plotted of binding energy per nucleon against mass number, we are effectively mapping out the stability of all the elements, starting from hydrogen.

As can be seen from the graph opposite, there's a discernible pattern: low mass number nuclei are relatively unstable as they have low values for the "BE per nucleon", but generally become more stable up to about iron (Fe), or nickel (Ni), where the "BE per nucleon" is highest.

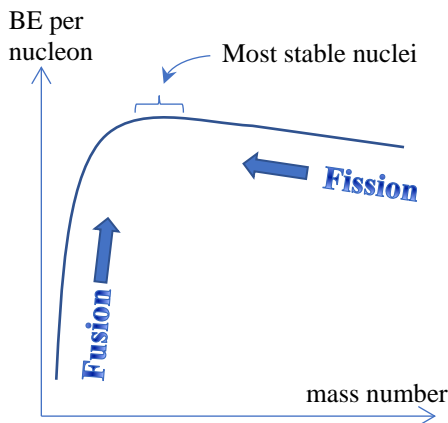
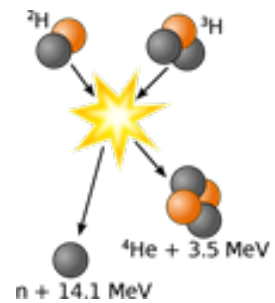
Above this (atomic number > ~60), the "BE per nucleon" decreases again, meaning that the nuclei heavier than iron/nickel become less and less stable.

This simple pattern is all we need to explain why energy is released by fusion **and** by fission.



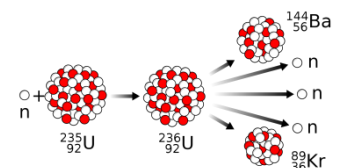
Fusion and Fission

Fusion is the process by which lighter elements join or fuse together to make heavier elements. It is the nuclei that fuse, and since nuclei are always positively charged, they will repel. This is why the lighter elements that are colliding during fusion must reach very high velocities (achieved by high temperatures) so that they get close enough to each other such that the strong nuclear force becomes stronger than the electrostatic repulsion.



Fusion releases energy only for low mass nuclei. This is explained by the BE per nucleon graph. 2 very light nuclei like ${}^2\text{H}$ and ${}^3\text{H}$ have low values for their BE per nucleon. When they fuse they make a heavier nucleus (He in this case) which is higher up the graph, i.e. has a higher value for the BE per nucleon. This means the nuclei are more tightly bound than before, and as discussed on page 27, this means the nuclei have slightly less mass than before, and so the loss in mass is **released as energy**.

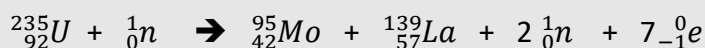
Fission is the process where a heavy nucleus splits up into 2 or more smaller fragments. In radioactive nuclei, this happens naturally (radioactive decay). If a large nucleus is bombarded with an external particle, e.g. a neutron, it can cause it to split into two 'daughter' nuclei of roughly equal masses. (This is known as induced fission and is the process used in nuclear power stations). By looking at the BE per nucleon graph, we can see that this leads to two lighter nuclei, both of which are higher up the graph, and hence, as with fusion, **energy is released**.



Calculating the energy released by Fusion and Fission reactions

When light nuclei fuse, or when heavy nuclei fission, more stable nuclei are created (their BE per nucleon increases). This means that the total mass of the products in both cases are slightly less than the reactants. This **mass defect** is converted into energy. The method for calculating the amount of energy released is very similar to the method for calculating binding energy, and assumes that **mass-energy is conserved**.

Example. Here's a typical fission reaction:



Calculate the energy released by 1kg of U-235. Mass of neutron = 1.00867 u;
Mass of ${}^{95}\text{Mo}$ = 94.906 u; Mass of ${}^{139}\text{La}$ = 138.906 u; Mass of ${}^{235}\text{U}$ = 235.044 u

Ignoring the mass of any electrons,

$$\text{MD} = (235.044 + 1.00867) - (94.906 + 138.906 + 2(1.00867)) = 0.06727\text{u}$$

$$\therefore \text{Energy released per reaction} = \text{MD} \times 931 = 62.63 \text{ MeV}$$

$$\text{Number of reactions} = \frac{\text{total mass}}{\text{mass of one U nucleus}} = \frac{1 \text{ (kg)}}{235.044 \times (1.66 \times 10^{-27})} = 2.56 \times 10^{24}$$

$$\text{Hence, total energy released (by 1 kg)} = 2.56 \times 10^{24} \times 62.63 \text{ MeV} = 1.60 \times 10^{23} \text{ MeV} (= 2.57 \times 10^{16} \text{ J})$$

Note!!

If masses are given in 'kg', use: $E = \text{MD} \times c^2$ (Answer in Joules, J)

Acknowledgements

Section	Image	Acknowledgements
Cover	Physics	CristiNistor / getty images
3.1	Roller coaster	Image by Coasterman1234 / CC BY 3.0
3.2	Guitar	Max Pixel http://bit.ly/2oM9Etu
3.2	Bouncy castle	This image appears to be in the public domain however if there are any omissions or inaccuracies please inform us so that any necessary corrections can be made resources@wjec.co.uk .
3.2	Rev-counter / Speed meter	Image by Zach Dischner / CC BY 2.0
3.2	Door mechanism	Image by Santeri Viinamaki / CC BY 4.0
3.2	Mountain bike	Pixabay http://bit.ly/2oP2g0g
3.2	Resonance curve 1	Graph by sjlegg / Wikimedia Creative Commons
3.2	Resonance curve 2	Graph by Geek3 / CC BY 3.0
3.2	The Millennium bridge, London	Image by Peter Trimming / CC BY 2.0
3.5	Radioactive Decay Graph	Image by BenRG / Wikimedia Creative Commons
3.5	Radioactive Decay; alpha, beta and gamma	PublicDomainPictures.net / CC BY 1.0 http://bit.ly/2oWCEhh
3.5	Half-life values graph	Unable to trace copyright, please contact us if you are the copyright holder.
3.5	Particles	Image by Starsandspirals / CC BY 2.0
3.5	Binding Energy	Image by Dirk Hünninger / CC BY 3.0
3.5	Binding Energy per nucleon	Image by Wikimedia Commons http://bit.ly/2FqooUQ
3.5	Fusion and Fission	Image by Wykis / Wikimedia Creative Commons
3.5	Fission reaction	Image by MikeRun / CC BY 4.0