

Definitions:

The **displacement** of a point B from a point A is the shortest distance from A to B, together with the direction as it is a vector.

$$\text{Mean speed} = \frac{\text{total distance travelled}}{\text{total time taken}} = \frac{\Delta x}{\Delta t}$$

Instantaneous speed = rate of change of distance.

$$\text{Mean velocity} = \frac{\text{total displacement}}{\text{total time taken}}$$

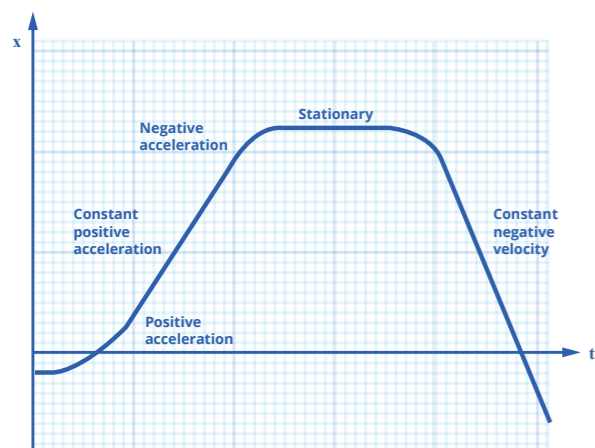
Instantaneous velocity = rate of change of displacement.

$$\text{Mean acceleration} = \frac{\text{change in velocity}}{\text{time taken}} = \frac{\Delta v}{\Delta t}$$

Instantaneous acceleration = rate of change of velocity.

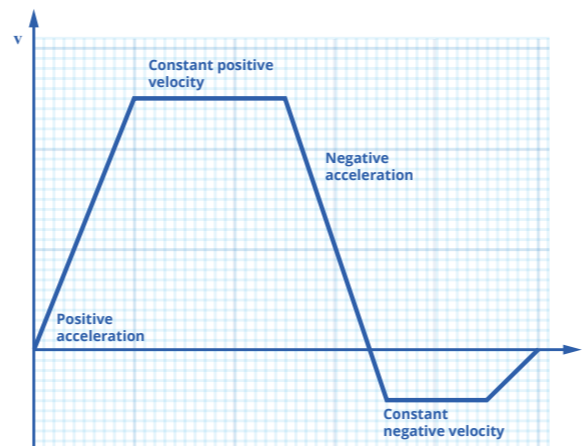
Note; displacement, velocity and acceleration are vectors.

x-t graphs:



The **gradient of the graph represents the velocity**, steeper lines mean a higher velocity. Using **tangents** to the curve and calculating their gradient allows you to calculate the velocity at any point.

v-t graphs:



The **gradient of the line is the acceleration** and the **area under the line is the displacement**. Note that the displacement may be negative.

Kinematic equations:

In addition to using the equations you must be able to derive them.

$$\mathbf{X.} \quad a = \frac{\Delta v}{\Delta t} = \frac{v-u}{t}$$

rearranging this gives $v = u + at$

1. Displacement = area under the line. The area, x , can be calculated using the trapezium rule.

$$x = \frac{1}{2}(u + v)t$$

2. The area can also be calculated by splitting the trapezium into a rectangle and a triangle.

$$x = ut + \frac{1}{2}t(v - u) \quad \text{substituting equation 1 into this gives}$$

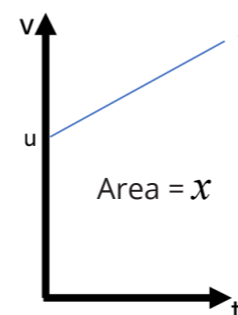
$$x = ut + \frac{1}{2}at^2$$

3. Substituting for t in equation 1 using equation 2 gives,

$$x = \frac{1}{2} \frac{(u + v)(u - v)}{a} \quad \text{Rearranging this equation gives,}$$

$$v^2 = u^2 + 2ax$$

Practise deriving these equations, showing every step of the algebra.



Falling under gravity:

Without air resistance, the acceleration due to gravity causes a constant acceleration downwards. Therefore, it can be used in the kinematic equations where $a = g = 9.81 \text{ m s}^{-2}$.

For example; the height reached by a cannonball fired vertically upwards at 40 m s^{-1} can be calculated using equation 4.

$$u = 40 \text{ m s}^{-1}$$

$$v = 0 \text{ m s}^{-1}$$

$$a = -9.81 \text{ m s}^{-2}$$

$$x = ?$$

$$0^2 = 40^2 + 2 \times -9.81 \times x$$

$$x = 82 \text{ m}$$

Note that a is negative as it is in the opposite direction to the initial velocity.

With air resistance, falling objects can reach their **terminal velocity**. This means that you cannot use the kinematic equations as the acceleration is not constant.

Projectiles:

An object, launched horizontally experiences a downward acceleration due to gravity. **This does not affect its horizontal motion but it will accelerate vertically downwards**, causing the object to move in a curve. The **time taken to fall is the same as if it was dropped from a stationary position**, and can be calculated using the kinematic equations.

If the object is **launched at an angle**, then you must **calculate the vertical and horizontal components** of the velocity and then apply any kinematic equations to those components.

You can then calculate the final horizontal and vertical components and add the vectors (see Unit 1.1) to calculate the final velocity.