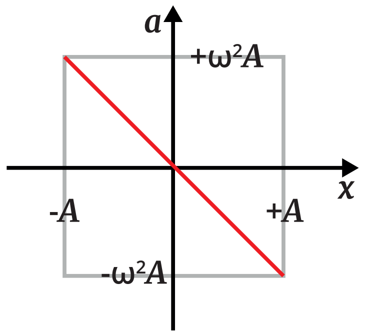
GCE Physics Unit 3.2 Vibrations

Simple harmonic motion (SHM):

Simple harmonic motion (SHM) occurs when an object moves such that **its acceleration is always directed toward a fixed point and is proportional to its distance from the fixed point**. This can be expressed in this equation:

$a = -\omega^2 x$

where $-\omega^2$ is a constant. Plotting a graph of a against x would give this shape, this is the same shape for all SHM.



A, is amplitude, the **maximum value of the displacement**.

Other key definitions are; the period, T, which is the **time taken for one complete cycle**, and frequency, f, which is the **number of oscillations per second**. (See unit 3.1)

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

Another way of defining SHM is where the motion of a point whose displacement, x, changes with time, t, according to $x = A\cos(\omega t + \varepsilon)$, where A, ω and ε are constants.

ε is the phase constant. It is normally 0 or $\pi/2$ depending on the displacement at time, t = 0.

If at t = 0 the displacement is at its maximum, then ε = 0 and if at t = 0, the displacement is 0, then $\varepsilon = \pi/2$.

The velocity during the motion can be calculated using the following equation:

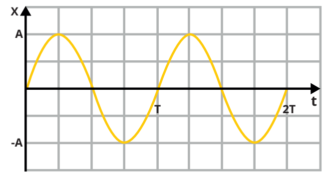
$v = -A\omega \sin(\omega t+\varepsilon)$

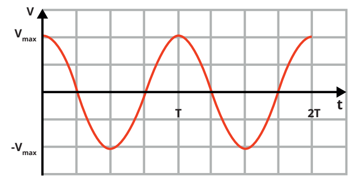
The maximum value for $\sin(\omega t+\varepsilon)$ = 1. Therefore, the maximum velocity, $v_{max} = A\omega$.

Graphical representation of SHM:

The equations show that both x and v vary sinusoidally with time during SHM.

Note that v is at a maximum when x = 0.



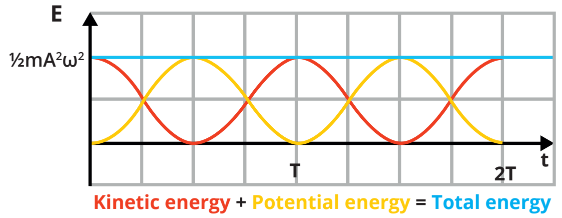


Energy:

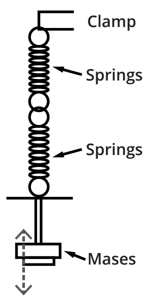
A body during SHM will have a constant energy, but it will transfer from potential energy to kinetic energy.

The kinetic energy of a body during SHM can be calculated using $E_k = \frac{1}{2}mv^2$ and therefore $E_{k\,max} = \frac{1}{2}mvA^2\omega^2$.

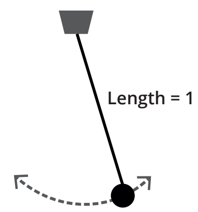
When the kinetic energy is at its maximum, the potential energy will be zero. Therefore, the energy of the body in SHM is always equal to $\frac{1}{2}mvA^2\omega^2$.



Two common examples of SHM are **masses on a spring** and a **simple pendulum**. The equations to calculate the period is given for each.



$$T = 2\pi\sqrt{\frac{m}{k}}$$



$$T = 2\pi\sqrt{\frac{l}{g}}$$

You must be able to describe using the apparatus to calculate k or g, as well as using the equations.

Free oscillations:

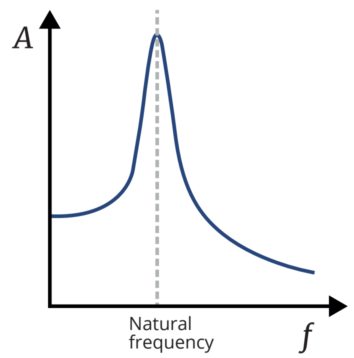
Free oscillations occur when an oscillatory system (such as a mass on a spring, or a pendulum) is displaced and released. The frequency of the free oscillations is called the system's **natural frequency**.

In reality, the oscillations will not stay at the same amplitude indefinitely but, will decrease over time due to resistive forces. This is known as **damping**.

Damping is very important in a number of situations. For example, in a car's shock absorbers, the vibration caused by going over a bump does not last long (otherwise all the passengers will feel unwell). A specific case is **critical damping**, where the resistive forces on the system are just large enough to **prevent oscillations** occurring at all when the system is displaced and released.

Resonance:

When a sinusoidally varying driving force is applied to an oscillating system, if the **frequency of the applied force is equal to the natural frequency** of the system, the amplitude of the resulting oscillations is large. This is **resonance**.



Resonance can be **useful**, for example, microwaves with a frequency similar to the natural frequency of water molecule vibration are used in microwave ovens to heat the water molecules.

However, it is **not always useful**. For example, the frequency of people walking on the millennium bridge was close to the natural frequency of the bridge and caused it to oscillate dangerously.

a = acceleration in m/s^{-2}
x = displacement in m
f = frequency in Hz
T = period in s
m = mass in kg
l = length in m
ω = angular velocity rad s^{-1}
A = amplitude in m
ε = phase constant in rad
t = time in s
k = spring constant in N m^{-1}