GCE Physics Unit 4.3 Orbits and the Wider Universe

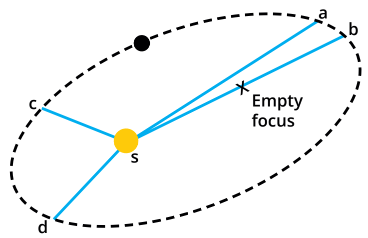
Kepler's laws:

There are 3 laws, known as Kepler's laws, to describe the motion of a body, such as a planet, in orbit.

1st law: each planet moves in an ellipse with the Sun at one focus.

2nd law: the line joining a planet to the centre of the Sun sweeps out equal areas in equal times.

Area abs = area cds



3rd law: T^2, the square of the period of the planet's motion, is proportional to r^3, where r is the mean distance from the planet to the star, when the orbit is close to circular.

The reason planets orbit the Sun is due to the **gravitational force** of the sun on the planet. Therefore, it is possible to calculate the **force between two bodies** in space using this equation. (See unit 4.2)

$$F = \frac{GM_1M_2}{r^2}$$

When the orbit is close to circular, when the mass of the object in orbit is much less than the mass of the object it is orbiting, the gravitational force is equal to the centripetal force. This can be used to derive an expression for Kepler's 3rd law.

$$F = \frac{GM_1M_2}{r^2} = \frac{M_2v^2}{r}$$

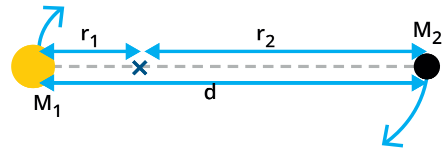
If the orbit is circular, then $v = \frac{2\pi r}{T}$ where T is the period of the orbit and $2\pi r$ is the circumference of the circle. Substituting this into the equation will give:

$$T^2 = \frac{4\pi^2r^3}{GM_1}$$

Practise the algebra to derive this equation.

Mutual orbits:

Most orbits are not circular, instead **both** objects **orbit a centre of mass between the two.**



The location of the centre of mass can be calculated using this equation:

$$r_1 = \frac{M_1}{M_1 + M_2}d$$

As the radius of the orbit and the distance between the objects are not equal, another equation must be used to calculate the orbital period.

$$T = 2\pi\sqrt{\frac{d^3}{G(M_1 + M_2)}}$$

Circular orbits:

A good approximation in most cases, as $M_1 >> M_2$, is

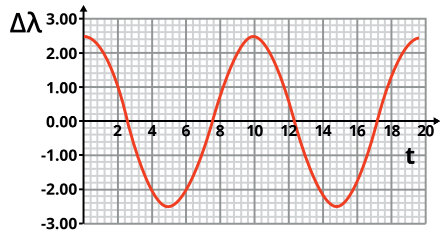
$$M_1 + M_2 \approx M_1$$

This approximation and understanding of circular motion, i.e $v = \frac{2\pi r}{T}$ can be used to calculate T to substitute into the above equation (or the reverse to calculate r). Remember that ***T* is the same for both** objects.

Doppler:

Observing the spectra of stars revealed that the wavelength of spectral lines changes. This is due to the apparent motion of the object. The velocity of the motion can be calculated using this equation.

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$



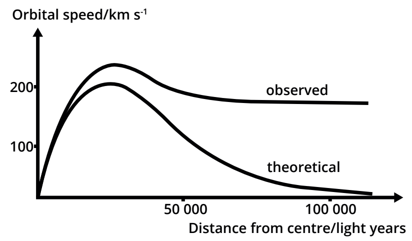
Study of the spectra from stars revealed $\Delta\lambda$ changes over time in a sinusoidal pattern. This can be used to calculate v at different points in the orbit, and calculate the **radial and the orbital velocity**. These can then be used in the above equations to calculate the radius of each orbit and the mass of the objects.

Dark matter:

The Doppler equation can be used to measure the speed of different parts of a rotating spiral galaxy.

Equating the gravitational force and the centripetal force can give this equation. $v = \sqrt{\frac{GM}{r}}$

The speed is therefore expected to decrease at larger distances from the centre of the galaxy. However, measurements and calculations using the Doppler equation gives these results.



As the observed speed does not decrease as expected, and $v \propto M$, M must increase as r increases. The extra mass is not visible and therefore is attributed to a cloud of dark matter surrounding the galaxy.

The recently discovered **Higgs boson** is linked to the mass of particles including dark matter.

The Hubble constant:

Measurements of the radial velocity of galaxies plotted against the distance to the galaxy gave a straight line.

$v \propto D$ the gradient of the line, the proportionality constant is known as the Hubble constant. $\boldsymbol{H_o}$ **= 2.2 × 10^{-18} s^{-1}**

This can be written as the following equation: $v = H_o D$

Comparing this to $v = \frac{x}{t}$ **gives an expression for the age of the universe as** $\frac{1}{H_o}$.

Critical density of the universe:

For a 'flat' universe, when time is infinite, the **radial velocity of the galaxies is equal to the escape velocity** (see unit 4.2).

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

The mass of the universe inside a sphere is $M = \frac{4}{3}\pi r^3 \times \rho_c$

and combining this and the equation $v = H_o D$ gives this expression for the critical density of the universe.

$$\rho_c = \frac{3H_o^{\ 2}}{8\pi G}$$

Practise the algebra to derive this equation.

F = force in N | r = radius in m | d = distance in m | λ = wavelength in m | H_o = Hubble constant

M = mass in kg | T = period of orbit in s | v = velocity in m s^{-1} | D = distance in m | ρ_c = critical density of the universe in kg m^{-3}