GCE Physics Unit 4 Option A, Alternating Currents

A rotating coil in a magnetic field generates an emf. The magnitude of the emf is given by Faraday's law (see Unit 4.5), which states the induced emf is equal to the rate of change of flux linkage:

$$\text{Flux linkage} = BAN\cos\theta$$

If the coil is rotating with angular velocity ω, then θ at time $t = \omega t$. This can be substituted into the equation to give:

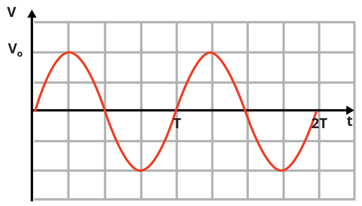
$$\textbf{Flux linkage} = BAN\cos\omega t$$

As the emf is the rate of change of flux linkage, this gives the following equation for the induced emf:

$$V = \omega BAN\sin\omega t$$

A rotating coil in a magnetic field will produce a sinusoidally alternating emf, with a **period of one cycle = T** and **frequency, the number of cycles per second = f**:

$$f = \frac{1}{T}$$



From the equation above, the maximum emf = ωBAN. This is **the peak voltage**, V_o. However, it is more practical to give the rms voltage for an alternating supply. This is the root mean square voltage, V, and is a way of expressing an average voltage:

$$V = \frac{V_o}{\sqrt{2}}$$

Similarly, current can be calculated:

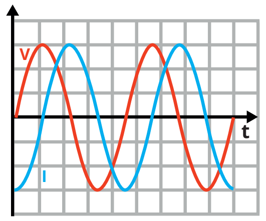
$$I = \frac{I_o}{\sqrt{2}}$$

To calculate the power dissipated in a circuit, **the rms values for current and voltage must be used**.

$$P = IV = I^2R = \frac{V^2}{R}$$

In an AC circuit, the resistors act in the same way as in a DC circuit (see unit 1.2 and 1.3). However, when a circuit contains an inductor or a capacitor, they act in a different way.

Inductor (L):



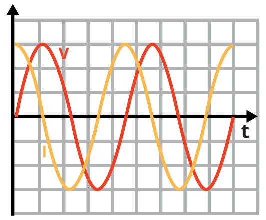
When the potential difference (p.d.) and the current through an inductor are plotted on the same graph, the current varies sinusoidally but **lags behind the p.d. by 90°**.

The reactance, X_L, is a measure similar to the resistance, of an inductor and it can be calculated using this equation:

$$X_L = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \omega L$$

Where $\omega = 2\pi f$ and f is the frequency of the alternating supply.

Capacitor (C):



The current through a capacitor **leads the p.d. by 90°**.

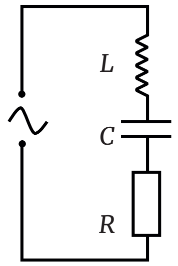
The reactance, X_C, of the capacitor can be calculated using this equation:

$$X_C = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{1}{\omega C}$$

In both capacitors and inductors, the **mean power dissipated is zero**, as they will both store energy and then release the energy during a cycle. This means that when calculating the power of an AC circuit with a resistor, inductor and a capacitor, the power dissipated will be equal to the power dissipated by the resistor only.

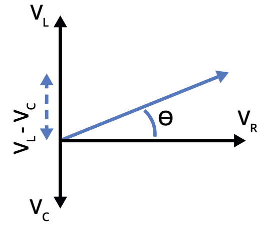
A **phasor**, similar to a vector, describes the magnitude and direction of a resultant p.d. or resistance.

RCL circuits:



Due to the difference in phase of the p.d. across the components, the p.d. can be represented in this diagram. The resultant p.d. can be calculated using Pythagoras' theorem,

$$V = \sqrt{V_R^{\,2} + (V_L - V_C)^2}$$



and the angle,

$$\theta = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right).$$

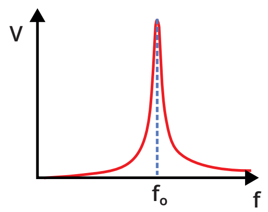
As the current through each component must be equal, due to conservation of charge, applying the equation $V = IR$ to each part allows the following equation to be derived to give a value for the impedance, Z, of the combination.

$$Z = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (\omega L - {}^{1}/_{\omega C})^2}$$

Note that these equations are valid for different combinations of R, L and C. i.e. RL circuits and RC circuits.

Resonance:

An RCL **circuit will be at resonance when $X_L - X_C = 0$**. At this point the impedance will be at a minimum = R.



Therefore, the resonance frequency of the circuit can be derived from this equation, $X_L = X_C$.

$$\boldsymbol{\omega L = \frac{1}{\omega C}}$$

As $\omega = 2\pi f$, this gives the following equation for the resonance frequency, f_o,

$$\boldsymbol{f_o = \frac{1}{2\pi\sqrt{LC}}}$$

An important property of RCL circuits is its Q factor. This relates to how sharp the resonance curve is.

$$Q = \frac{V_{L\text{rms}}\text{ at resonance}}{V_{R\text{rms}}\text{ at resonance}} \quad \text{therefore} \quad Q = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR} \quad \text{as, at resonance,}$$

$V_{L_{rms}} = V_{C_{rms}}$ and $X_L = X_C$.

B = magnetic flux density in T; N = number of turns; t = time in s; V = potential difference in V; L = inductance in Henry (H); C = capacitance in F; Z = impedance in Ω

A = area in m²; ω = angular velocity; I = current in A; R = resistance in Ω; Q = Q factor; T = period in s; f = frequency in Hz; X_L = reactance in Ω; X_C = reactance in Ω