Constructing and Measuring

Year 7

#MathsEveryoneCan

2019-20

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Algebraic Thinking**						**Place Value and Proportion**					
	Sequences		Understand and use algebraic notation		Equality and equivalence		Place value and ordering integers and decimals			Fraction, decimal and percentage equivalence		
Spring	**Applications of Number**						**Directed Number**			**Fractional Thinking**		
	Solving problems with addition & subtraction		Solving problems with multiplication and division			Fractions & percentages of amounts	Operations and equations with directed number			Addition and subtraction of fractions		
Summer	**Lines and Angles**						**Reasoning with Number**					
	Constructing, measuring and using geometric notation			Developing geometric reasoning			Developing number sense		Sets and probability		Prime numbers and proof	

Summer 1: Lines and Angles

Weeks 1 to 3: Construction, measurement and notation

Students will build on their KS2 skills using rulers, protractors and other measuring equipment to construct and measure increasingly complex diagrams using correct mathematical notation. This will include three letter notation for angles, the use of hatch marks to indicate equality and the use of arrows to indicate parallel lines. Pie charts will be studied here to gain further practice at drawing and measuring angles.

National curriculum content covered:

- use language and properties precisely to analyse 2-D shapes
- begin to reason deductively in geometry including using geometrical constructions
- draw and measure line segments and angles in geometric figures, including interpreting scale drawings
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right-angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- use the standard conventions for labelling sides and angles
- construct and interpret pie charts for categorical, ungrouped and grouped numerical data
- Identify and construct triangles

<u>Interleaving/Extension of previous work</u>

- revisit four operations

Weeks 4 to 6: Geometric reasoning

This block covers basic geometric language, names and properties of types of triangles and quadrilaterals, and the names of other polygons. Angles rules will be introduced and used to form short chains of reasoning . The higher strand will take this further, investigating and using parallel line rules.

National curriculum content covered:

- use language and properties precisely to analyse 2-D shapes,
- begin to reason deductively in geometry including using geometrical constructions
- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right-angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- use the standard conventions for labelling sides and angles
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- apply angle facts, triangle similarity and properties of quadrilaterals to derive results about angles and sides, and use known results to obtain simple proofs
- understand and use the relationship between parallel lines and alternate and corresponding angles **(H)**
- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons **(H)**

<u>Interleaving/Extension of previous work</u>

- forming and solving linear equations
- revisiting addition and subtraction, including decimals

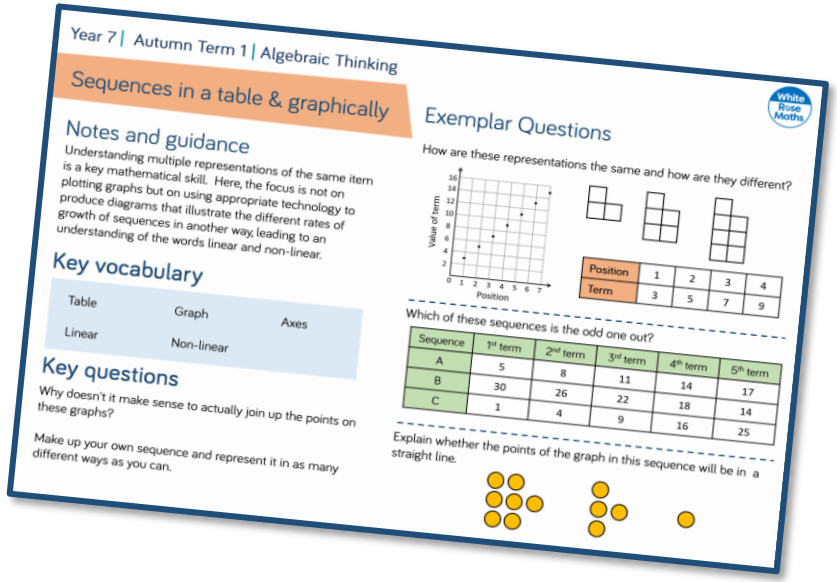
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



Year 7 | Autumn Term 1 | Algebraic Thinking

Sequences in a table & graphically

Notes and guidance

Understanding multiple representations of the same item is a key mathematical skill. Here, the focus is not on plotting graphs but on using appropriate technology to produce diagrams that illustrate the different rates of growth of sequences in another way, leading to an understanding of the words linear and non-linear.

Key vocabulary

Table, Graph, Axes, Linear, Non-linear

Key questions

Why doesn't it make sense to actually join up the points on these graphs?

Make up your own sequence and represent it in as many different ways as you can.

Exemplar Questions

How are these representations the same and how are they different?

Position	1	2	3	4
Term	3	5	7	9

Which of these sequences is the odd one out?

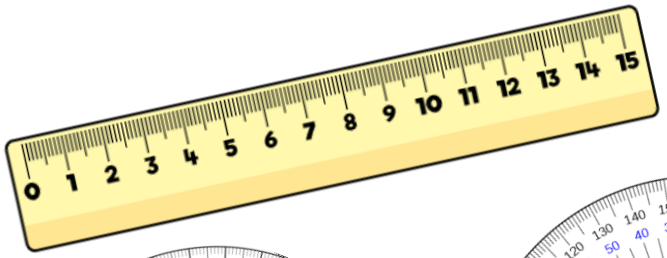
Sequence	1st term	2nd term	3rd term	4th term	5th term
A	5	8	11	14	17
B	30	26	22	18	14
C	1	4	9	16	25

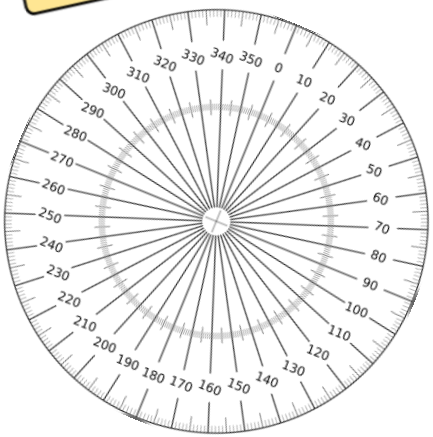
Explain whether the points of the graph in this sequence will be in a straight line.

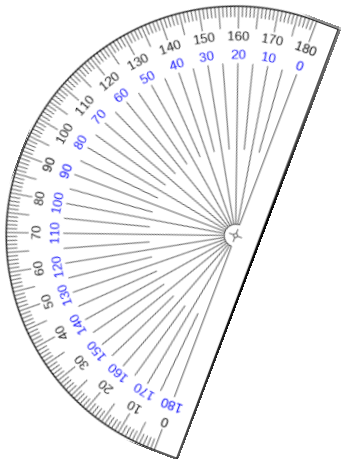
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol  .
- For each block, we also provide ideas for key representations that will be useful for all students.

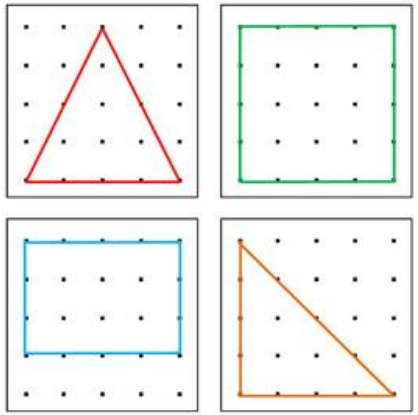
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

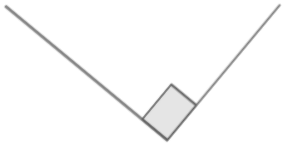
Key Representations

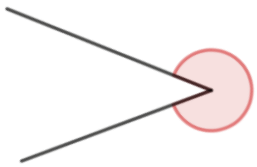


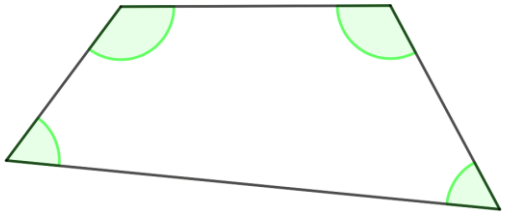


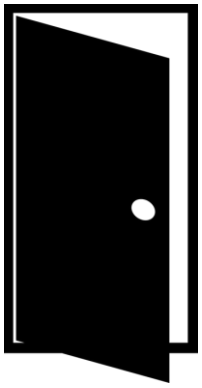


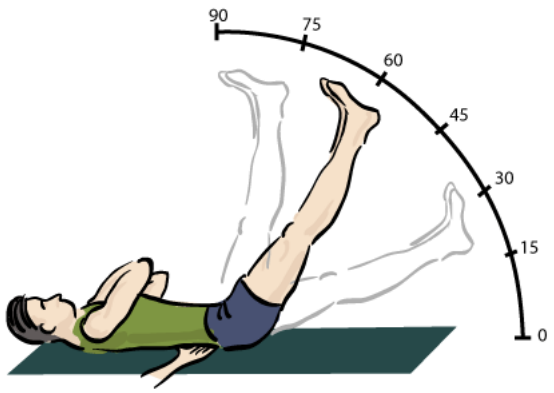












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas of equipment and representations that you might use during Construction and Measuring.

Opening and closing a door or a book allows students to visualise angles as a turn. They can also physically turn themselves and use their arms and legs to demonstrate angles and turns as in the lower right diagram.

Pictorial representations should be shown in a variety of orientations to avoid misconceptions such as angles always being measured from the horizontal.

Construction and Measuring

Small Steps

- Understand and use letter and labelling conventions including those for geometric figures
- Draw and measure line segments including geometric figures
- Understand angles as a measure of turn
- Classify angles
- Measure angles up to 180°
- Draw angles up to 180°
- Draw and measure angles between 180° and 360°
- Identify perpendicular and parallel lines
- Recognise types of triangle
- Recognise types of quadrilateral

Construction and Measuring

Small Steps

- Identify polygons up to a decagon
- Construct triangles using SSS
- Construct triangles using SSS, SAS and ASA
- Construct more complex polygons
- Interpret simple pie charts using proportion
- Interpret pie charts using a protractor
- Draw pie charts

Letter and labelling conventions

Notes and guidance

Students need to be able to describe a line segment and geometric figures using letter notation. They should always use a capital letter to define a vertex and know that two letters are required to define a line segment and three letters for an angle. Polygons should be described by naming the vertices cyclically and often but not always in alphabetical order.

Key vocabulary

Line	Line segment	Geometric figure
Notation	Polygon	

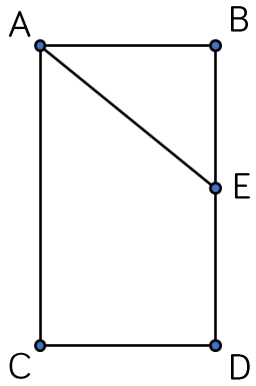
Key questions

How many points do you need to define a straight line?

How many points do you need to define a polygon?

Exemplar Questions

A geometric figure is shown to the right.

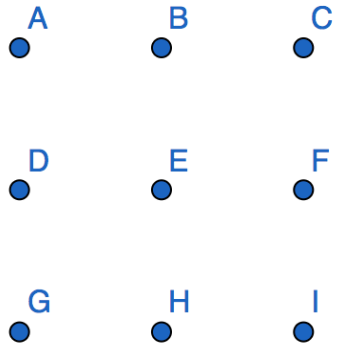


Use letter notation to fill in the blanks in the statements below.

Angle _____ is a right-angle.
Line segments ____ and ____ are vertical.
Shape _____ is a triangle.
Shape _____ is a quadrilateral.

Draw a trapezium and label the vertices KLMN.
Draw a line segment from point L to point N.
Use three letter notation to identify the triangles formed within the trapezium.

How many squares can you construct using the points labelled?
Use letter notation to name each one.



Draw and measure line segments

Notes and guidance

During this small step students will measure line segments within geometric figures to an accuracy of up to 1 mm. Students should be expected to convert freely between metric units. They should measure objects within the classroom and the wider environment and be able to justify the unit used.

Key vocabulary

Line	Segment	Length
Height	Width	Figure

Key questions

What is the difference between a line and a line segment?

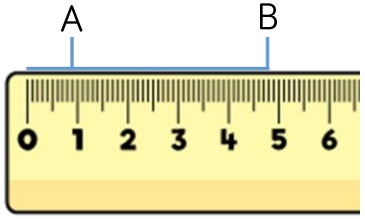
What would you measure in millimetres/centimetres etc.?

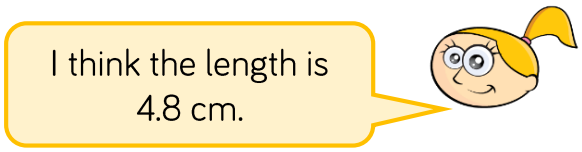
Exemplar Questions

WXYZ is a square. Side XY is 5cm long.

Draw the square and find the lengths of its diagonals WY and XZ.
Is there more than one way to draw this square?
Investigate if WXYZ were a rhombus instead.

Eva is measuring the length from point A to point B.





What mistake has Eva made? What advice would you give her?

Line A is $4\frac{1}{4}$ cm long. Line B is 70 mm shorter than line A.
Lines C and D are drawn accurately below.
Put these lines in ascending length order.

Angles as a measure of turn

Notes and guidance

This step ensures that students understand that angles are a measure of turn. They will understand that an angle is formed by two lines meeting at a point. A variety of language should be used to describe the size and direction of a turn. Demonstrations of angles as a turn such as opening or closing a door, and the angle formed at the elbow should be shown to students.

Key vocabulary

Quarter/Half/Three Quarter/Full Turn

Degrees Angles Rotation

Key questions

How can we measure the size of a turn?

How can we describe the direction of a turn?

Does direction matter for a turn of 180°?

Exemplar Questions

A diver performs a dive with two and a half-turns.
How many degrees do they rotate through?

A ship is sailing North. It turns to face east.
Mo and Rosie are discussing how far the ship has turned.

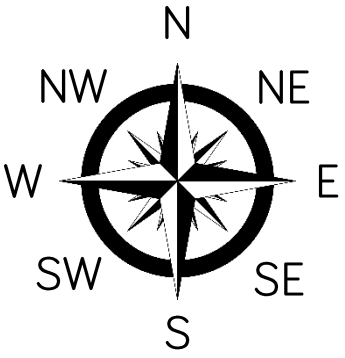
I think the ship has turned one quarter turn clockwise.

I think the ship has turned 270°

Who do you agree with? Why?

Write down three things in your classroom which turn through an angle.

Sam starts by facing North.
She turns clockwise to face West.
How many degrees has she turned through?



Asif is facing East.
He turns anticlockwise through 540°
Which direction is he now facing?

Alicia starts by facing South East.
She turns clockwise through 270°
Which direction is she now facing?

Classify angles

Notes and guidance

After completing this step students should be able to classify angles by sight, including within geometric figures. They should be familiar with and able to use conventional markings for right-angles. Students should also be introduced to the vocabulary of interior and exterior angles.

Key vocabulary

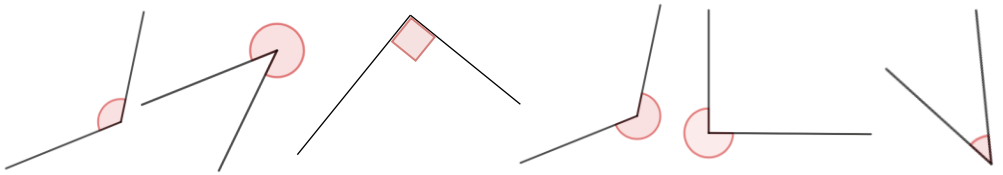
Angle	Acute	Obtuse	Right-angle
Reflex	Interior	Exterior	

Key questions

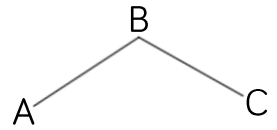
How do we illustrate that an angle is 90°?
How do we know which angle we are measuring?
Will turning through two acute angles result in turning through an obtuse angle?
Will turning through two obtuse angles result in turning through a reflex angle?

Exemplar Questions

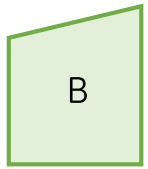
Classify the angles as acute, obtuse, reflex or right-angles.

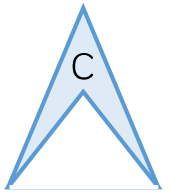


Amir says angle ABC is obtuse.
Whitney says angle ABC is a reflex angle.
Who is correct and why?



Complete the table about the interior angles for each of the shapes.





Shape	Acute angles	Obtuse angles	Reflex angles	Right-angles
A				
B				
C				

Repeat for the exterior angles of each shape.

Measure angles up to 180°

Notes and guidance

Students use a protractor graduated in degrees to measure angles up to 180° including within geometric figures. Accuracy of measurement should be within a degree. Students should estimate the size of angles before measuring by comparing them to 90° and 180°. Students could estimate and measure angles in the real world such as the angle between the hands of a clock.

Key vocabulary

Protractor	Degrees	Right-angle
Half-turn	Sum	Measure

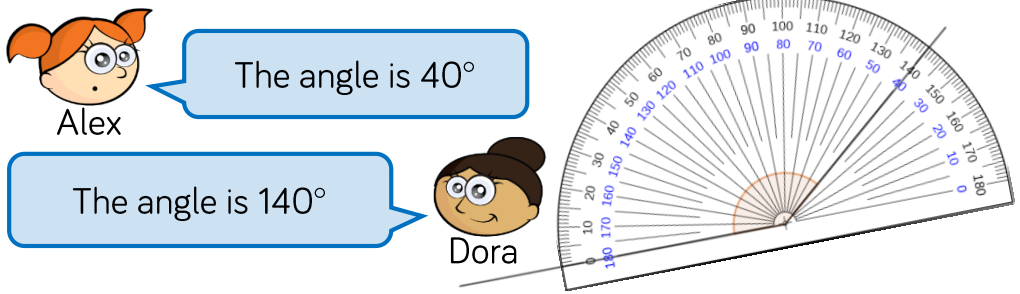
Key questions

How do we know which scale should be used to measure the angle?

How do we know where to put the protractor when measuring an angle?

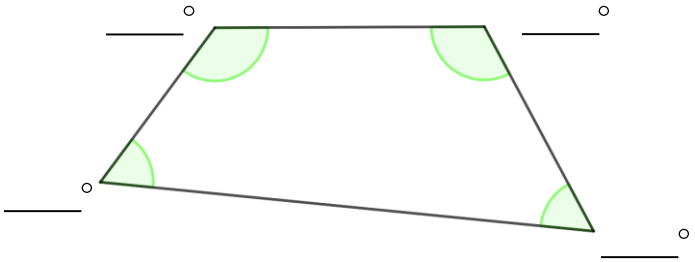
Exemplar Questions

Alex and Dora measure the angle using a protractor.



Who do you agree with? Why?

Measure the size of each of the interior angles in the shape.



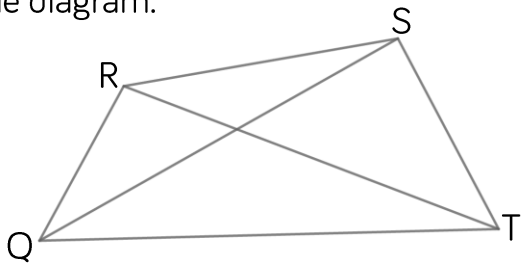
Complete the statements about the diagram.

∠RTS is _____°

∠QRS is _____°

The sum of angles SRT and QRT is _____°

The difference between angles QST and SQT is _____°



Draw angles up to 180°

Notes and guidance

In this small step, students draw angles up to 180° using a ruler and protractor. They should be able to construct the angle either at a specified point on a line or at the end of a line segment. As with measurement, angles should be drawn to an accuracy of within 1 degree. When checking their drawings, students should make comparisons with 90° and 180°

Key vocabulary

Angle	Protractor	Construct

Key questions

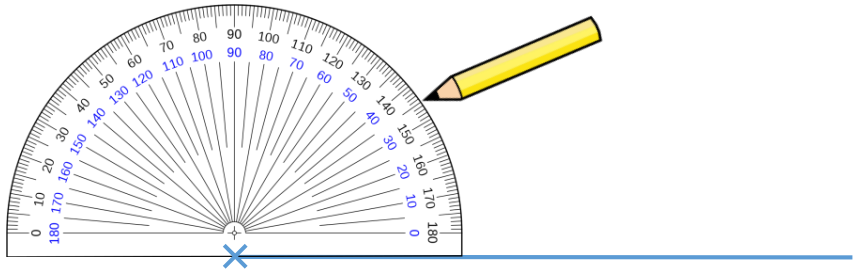
How do you choose which scale to use on a protractor?

Is it possible to draw an angle of 180°?

Why are there two scales on a protractor?

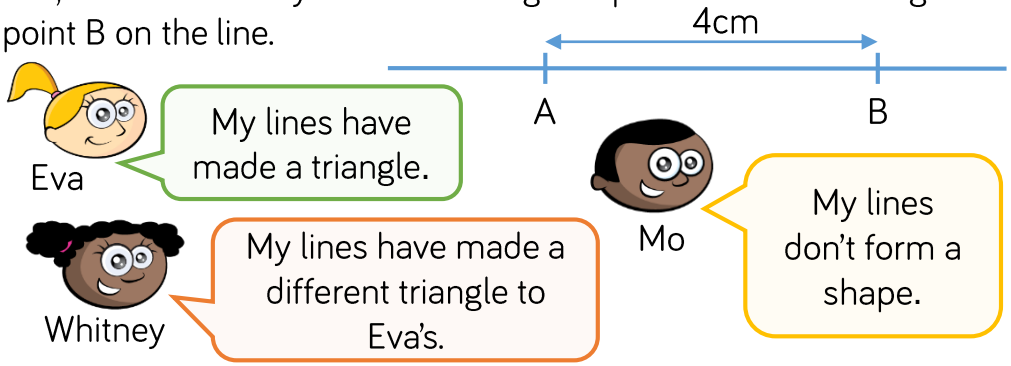
Exemplar Questions

Teddy is drawing an angle of 35°
He marks the angle with his pencil as shown.



Will Teddy's diagram show the correct angle? How do you know?

Eva, Mo and Whitney draw a 63° angle at point A and a 15° angle at point B on the line.



Who is correct? Demonstrate your reasoning by drawing a diagram.

In each diagram, measure and draw an angle of 115° at point A.

Draw and measure 180° to 360°

Notes and guidance

Students develop their skills acquired during the previous steps by drawing and measuring angles between 180° and 360°

Accuracy should be within one degree when drawing and measuring angles.

Key vocabulary

Angle	Protractor	Construct

Key questions

How many degrees are there in a full turn?

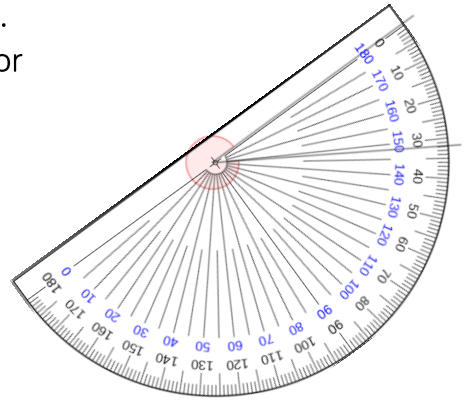
How can we use a protractor that doesn't go up to 360° to draw and measure angles over 180°?

Exemplar Questions

Measure all four angles shown.

Draw an angle of 258° at point A. How many ways can you do this?

Circle the correct size of the angle.
What mistakes have been made for the other answers?



A	B	C	D
32°	148°	322°	212°

Perpendicular and parallel lines

Notes and guidance

Students need to be able to identify parallel and perpendicular lines, including within geometric figures. They should use the correct notation to show where they have been identified. Examples of parallel and perpendicular lines in the real world should be explored.

Key vocabulary

Parallel Perpendicular Intersect

Key questions

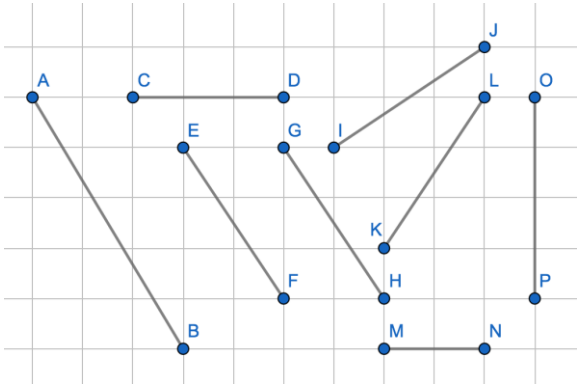
When are two or more lines parallel?

When are two lines perpendicular?

Can curved lines be parallel?

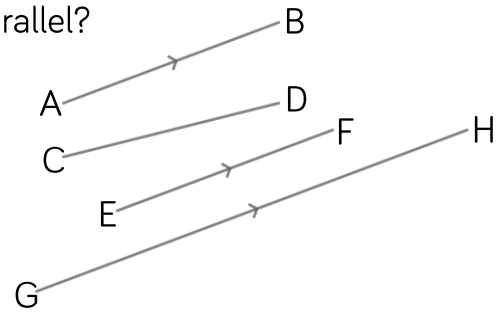
Exemplar Questions

Complete the sentences using two letter notation for each line segment.
CD is parallel to ___
IJ is perpendicular to ____
EF is neither parallel nor perpendicular to ____



Eva thinks that the lines are parallel.
Is she right?

Which of the line segments are parallel?



Tommy says the lines can't be perpendicular because they don't cross.
Do you agree?

Recognise types of triangles

Notes and guidance

Students should be able to recognise different types of triangles. They will be familiar with the properties of triangles from the primary curriculum. Students should be able to measure lengths and angles in triangles in order to classify them.

Key vocabulary

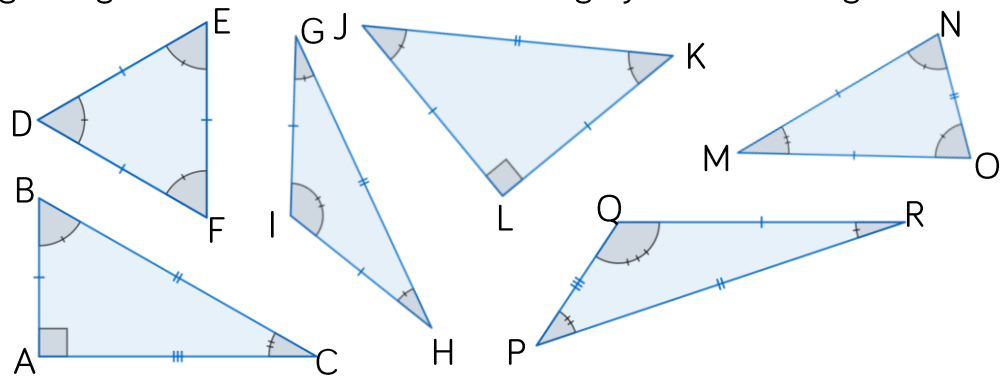
Equilateral	Isosceles	Scalene
Right-angled	Length	Angle

Key questions

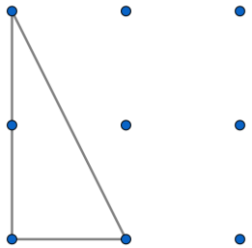
What is the difference between a scalene and an isosceles triangle?
What would you need to know about a triangle to be sure that it was equilateral?
Which types of triangle can also be right-angled?
Is an equilateral triangle also an isosceles triangle?

Exemplar Questions

Classify the following triangles as equilateral, isosceles, scalene or right-angled. Is there more than one category for each triangle?

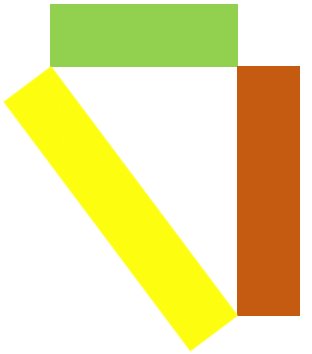


Make as many different triangles as possible using a 3 by 3 geoboard. Classify each triangle.
One example is shown.
Are there any types of triangle you cannot make?



Use Cuisenaire rods to construct triangles using one piece per side.
What types of triangles can you create?
Are there any types that you can't create?
Can you generalise?

Record your results in a table.



Recognise types of quadrilaterals

Notes and guidance

Students will be familiar with the vocabulary of quadrilaterals from key stage 2. This step revisits and consolidates their understanding. Students should be able to fluently distinguish between types of quadrilateral using appropriate terminology to justify their decisions.

Key vocabulary

Square	Rectangle	Kite	Rhombus
Parallelogram	Trapezium	Parallel	Perpendicular

Key questions

What property does every quadrilateral share?
Is a quadrilateral a polygon?
Which quadrilaterals always have an interior right-angle?
Which quadrilaterals sometimes have an interior right-angle?
Explain why a square is a rectangle and a parallelogram.

Exemplar Questions

Create a 4 by 4 square on your geoboard.
By moving only one vertex, which of the following can you make?

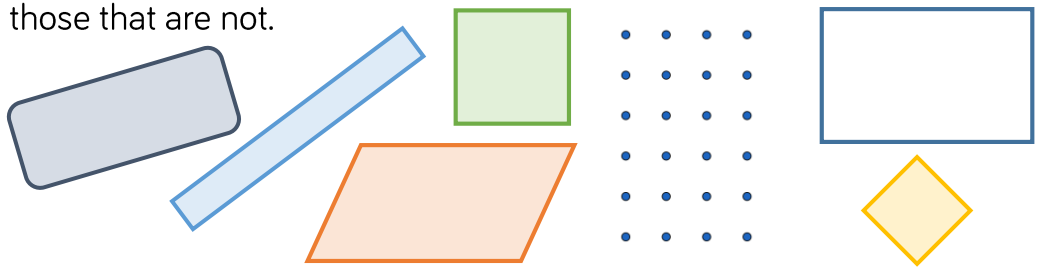
Kite	Trapezium	Parallelogram	Rhombus

Create a 4 by 6 rectangle on your geoboard.
By moving only one vertex, which of the following can you make?

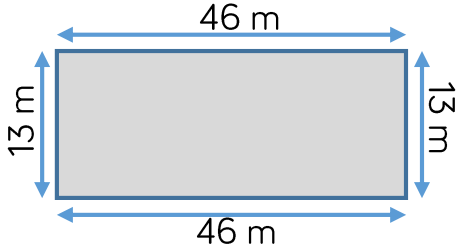
Kite	Trapezium	Parallelogram	Rhombus

How do your answers change if you're allowed to move 2 vertices?

Sort the shapes into two groups, those that are rectangles and those that are not.



A plan of a building is shown.
What else does Jack need to check to show that he is correct?



The building is in the shape of a rectangle because the opposite sides are equal in length.

Identify polygons up to a decagon

Notes and guidance

Students should identify polygons up to a decagon. They need to be able to distinguish between regular and irregular polygons. Students often think shapes are regular when their side lengths are equal without considering interior angles. They should relate vocabulary with other areas of mathematics as well as in the real world. For example, associating 'dec' with decimal and decathlon.

Key vocabulary

Polygon	Edges	Vertices	Angles
Equal	Length	Triangle	Decagon

Key questions

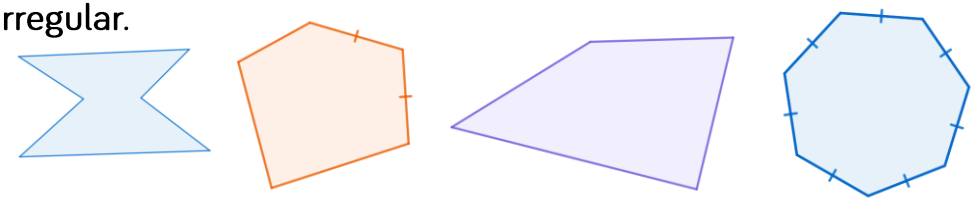
When is a polygon regular?

What name do we give to a regular three sided polygon?

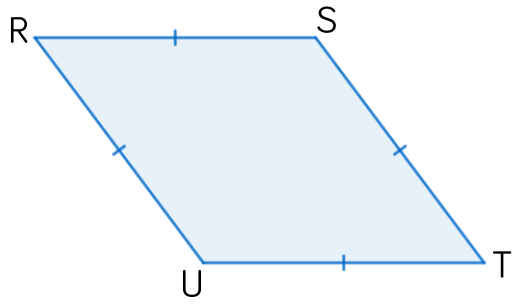
What name do we give to a regular four sided polygon?

Exemplar Questions

Write the name of each polygon and decide whether it is **regular** or **irregular**.

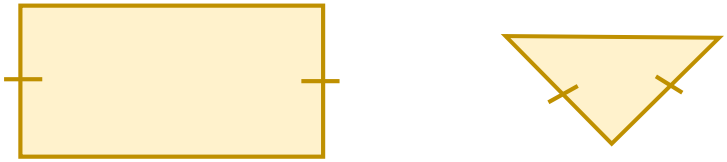


Jack says that the shape, RSTU, is a regular polygon.



Explain Jack's mistake.

How many types of polygon can be made by attaching the following two shapes at their edges?



Construct triangles – SSS

Notes and guidance

Students need to understand how to construct a triangle where all 3 sides are given. You may want students to try and do this using just a ruler and pencil at first as this will highlight that this is inaccurate. They should realise that it is more accurate to use a compass. Students should be able to explain why a certain set of side lengths will not make a triangle.

Key vocabulary

Pair of Compasses		Construct	
Side	Edge	Vertex	Point

Key questions

Is it possible to accurately construct a triangle given the side lengths using just a pencil and ruler?

Why is more accurate to use a pair of compasses?

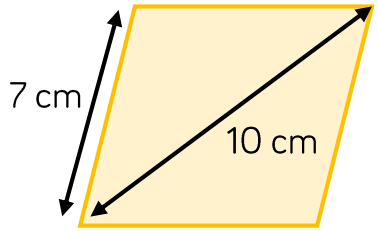
Exemplar Questions

Construct a line segment AB 10 cm long.
Plot point C such that AC and BC have lengths 8 cm and 5.5 cm respectively.
Measure the angle at each vertex and classify the triangle.
What equipment did you use?
Why is it more accurate to use a pair of compasses?

Construct a triangle with side lengths 6 cm, 8 cm and 10 cm.
What do you notice about this triangle?

Explain why you cannot construct a triangle with side lengths 4 cm, 5 cm and 11 cm.
Can you come up with another set of side lengths that will not make a triangle?
How do you know?

Here is a rhombus with side lengths of 7 cm and a longest diagonal of 10 cm.



Use a pair of compasses to construct a rhombus with sides of 5 cm and a longest diagonal of 8 cm.

Construct triangles – SSS, SAS, ASA

Notes and guidance

Students need to be familiar with the phrases side-side-side, side-angle-side and angle-side-angle. They should also understand why they represent the minimum information to draw a distinct triangle. They should be exposed to ambiguous cases when it is possible to construct two distinct triangles from the information given.

Key vocabulary

Isosceles	Equilateral	Scalene	Right-angled
Side	Edge	Vertex	Point

Key questions

Is it possible to construct a unique triangle given all three angles?

Why is it sometimes possible to draw two distinct triangles when given an angle and the length of two sides?

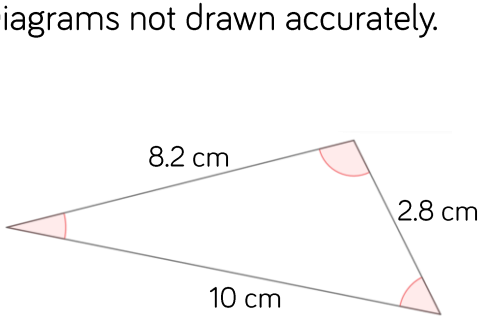
Exemplar Questions

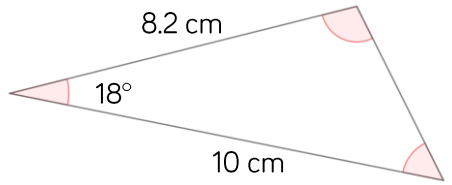
Construct a triangle WXY such that angle WXY is 40° and lengths WX and WY are integers that add to make 13 cm.
How many unique triangles can you construct?
Explain why this is possible.

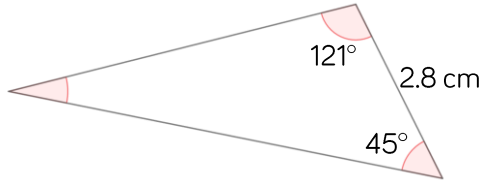
Which of the triangles are identical?

You must draw each triangle accurately and give reasons for your answer

Diagrams not drawn accurately.







Describe three different ways to construct an equilateral triangle with perimeter 189 mm.

Construct more complex polygons

Notes and guidance

Students should be able to draw more complex polygons and diagrams constructed using multiple polygons. This step is an opportunity to recap perimeter.
Letter notation should continue to be used for line segments, polygons and angles.

Key vocabulary

Polygon	Regular	Side	Vertex
Vertices	Rhombus	Diagonals	Compound

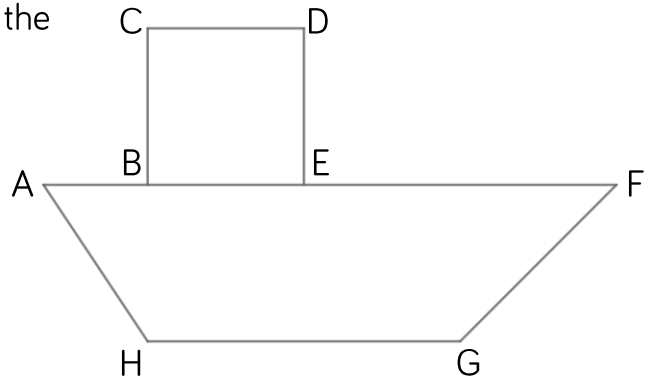
Key questions

Is it possible to construct an irregular polygon with equal angles?

Is it possible to construct an irregular polygon with equal side lengths?

Exemplar Questions

The boat has been constructed using 2-D shapes.
BCDE is a square with side length 3 cm.
Line segment BA is 2.4 cm long.
Point H is 3 cm directly beneath point B.
Line segment BF is 8.8 cm long.
Angle EFG is 45°
Construct an accurate diagram of the boat and complete the information below.



Length AH = ___

∠AHG = ___

Perimeter = ___

The height of the boat is ___.

Construct a rhombus EFGH with side length 5.7 cm and angle FGH measuring 53°
Measure and state the lengths of the diagonals using two-letter notation.

Can you construct a regular hexagon with side lengths 7 cm without using a protractor?

Interpret pie charts using proportion

Notes and guidance

In this small step students will interpret pie charts divided into equal portions, given the whole or part of the total frequency. Students should be able to make comparisons between multiple pie charts. Students should acknowledge that although they can compare proportions, this does not necessarily mean they can compare frequencies.

Key vocabulary

Proportion	Frequency	Fraction
Total	Comparison	Sector

Key questions

What do pie charts show us?

If two parts of the pie chart are the same size, what does that tell us?

If one part of two different pie charts are the same size, do they represent the same frequency?

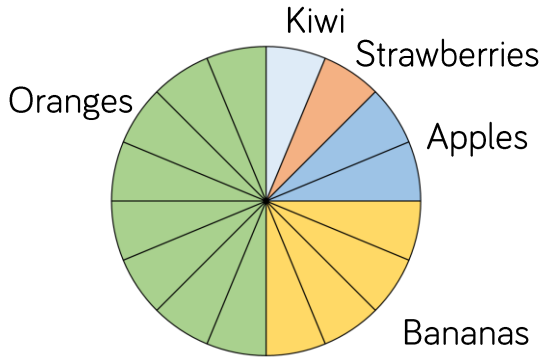
Exemplar Questions

The pie chart below shows the favourite fruits of a class. There are 32 students in the class.

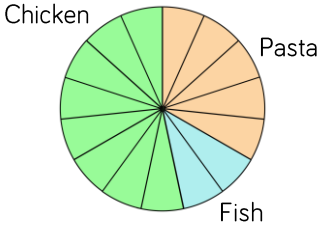
What fraction prefer bananas?

How many more students prefer oranges to apples?

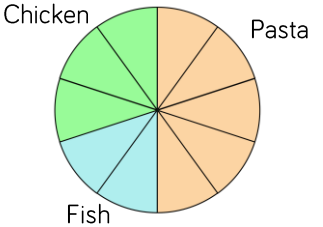
What was the least popular fruit?



The pie charts below show what students at two different schools chose for their lunch one day. Do you agree with the statements?



Fawton High School

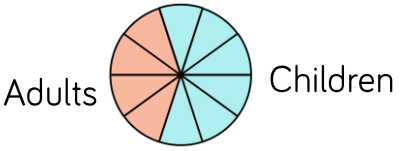


Burton Primary School

Half of the children at Burton Primary chose pasta.

More children chose pasta at Fawton High than at Burton Primary.

The pie chart shows the proportions of adults and children attending a musical. 240 children attended the musical. How many people attended the musical altogether?



Interpret pie charts using a protractor

Notes and guidance

Students will extend the skills developed in the previous step to interpret pie charts given the angles for each section.

Students should be familiar with a full turn being 360°

Key vocabulary

Protractor	Proportion	Frequency
Angle	Degrees	Sector

Key questions

If two pie charts are identical, do they represent identical frequencies?

What if the angle measured is between two marks on your protractor?

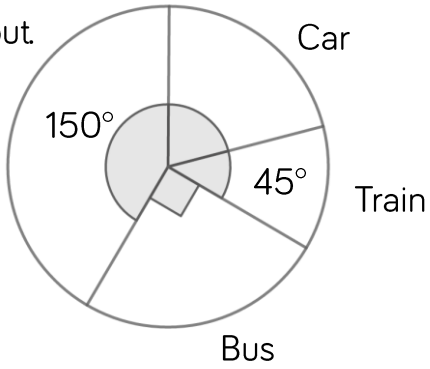
Exemplar Questions

The pie chart shows the proportion of how a group of employees travel to work. There are 24 employees.
Explain how the following were worked out.

"a quarter of the people go by bus. So 6 people go by bus."

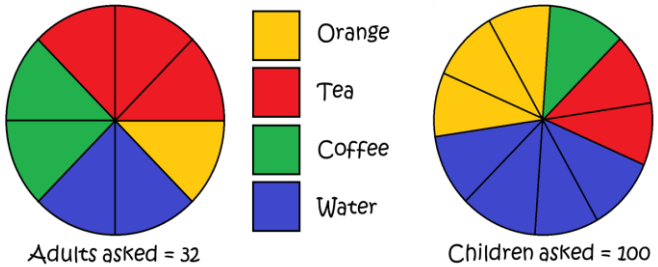
$\frac{150}{360} \times 24 = 10$
So 10 people walk.

$\frac{45}{360} = \frac{1}{8}$
One eighth of 24 is 3

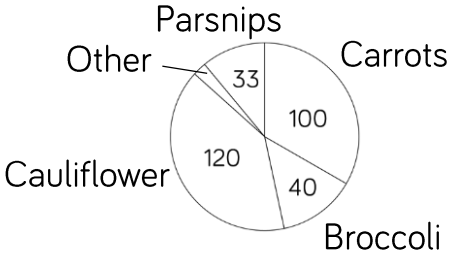


How many people came by car?
How many ways could you work this out?

The pie charts shows favourite drinks.
What fraction of adults prefer tea?
Which group has the most tea drinkers?



300 students were asked to name their favourite vegetable given the options carrots, broccoli, cauliflower, parsnips and other. How many chose other? Which vegetable was chosen by 40% of the students?



Draw pie charts

Notes and guidance

Students should be able to draw a pie chart given a complete or incomplete frequency table.

Students should be encouraged to consider whether a pie chart is the most appropriate representation for given data.

Key vocabulary

Protractor	Proportion	Frequency
Angle	Degrees	Sector

Key questions

What do you do when the total frequency is neither a multiple nor a factor of 360?

How do you calculate the angle of a sector of a pie chart?

Exemplar Questions

The table contains information about the cars in a car park.
Complete the table and draw a pie chart to represent the data.

Manufacturer	Frequency	Angle of sector
Volvo	10	
BMW	8	
Ford	16	
Kia	11	

The table contains information about the colour of students eyes in class 8a. There are 30 students in class 8a. Complete the table and draw a pie chart to represent the data.

Manufacturer	Frequency	Angle of sector
Brown	6	
Hazel	8	
Blue		60°
Green		48°
Silver	2	
Amber		

Tahir is drawing a pie chart using the frequency table.
Tahir's calculations are in red. Explain the mistake Tahir has made and then construct an accurate pie chart.

Sport	Frequency
Hockey	20
Rugby	28
Football	16
Basketball	26

$0.25 \times 20 = 5°$
$0.25 \times 28 = 7°$
$0.25 \times 16 = 4°$
$0.25 \times 26 = 6.5°$

$20 + 28 + 16 + 26 = 90$ $90 \div 360 = 0.25$