Angles in parallel lines and polygons

Year 8

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability
Spring	**Algebraic techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages			Standard index form	Number sense	
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons			Area of trapezia and circles		Line symmetry and reflection	The data handling cycle				Measures of location	

Summer 1: Developing Geometry

Weeks 1 and 2: Angles in parallel lines and polygons

This block builds on KS2 and Year 7 understanding of angle notation and relationships, extending all students to explore angles in parallel lines and thus solve increasingly complex missing angle problems. Links are then made to the closely connected properties of polygons and quadrilaterals. The use of dynamic geometry software to illustrate results is highly recommended, and students following the Higher strand will also develop their understanding of the idea of proof. They will also look start to explore constructions with rulers and pairs of compasses. This key block may take slightly longer than two weeks and the following blocks may need to be adjusted accordingly.
National Curriculum content covered includes:

- apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles
- understand and use the relationship between parallel lines and alternate and corresponding angles
- derive and use the sum of angles in a triangle and use it to deduce the angle sum in any polygon, and to derive properties of regular polygons
- use the standard conventions for labelling the sides and angles of triangle ABC
- derive and illustrate properties of triangles, quadrilaterals, circles, and other plane figures [for example, equal lengths and angles] using appropriate language and technologies
- derive and use the standard ruler and compass constructions (H only)

Weeks 3 and 4: Area of trapezia and circles

Students following the Higher strand will have met the formulae for the area of a trapezium in Year 7; this knowledge is now extended to all students, along with the formula for the area of a circle.

A key aspect of the unit is choosing and using the correct formula for the correct shape, reinforcing recognising the shapes, their properties and names and looking explicitly at compound shapes.
National Curriculum content covered includes:

- derive and apply formulae to calculate and solve problems involving: perimeter and area of triangles, parallelograms, trapezia
- calculate and solve problems involving: perimeters of 2-D shapes (including circles), areas of circles and composite shapes

Weeks 5 and 6: Line symmetry and reflection

The teaching of reflection is split from that of rotation and translation to try and ensure students attain a deeper understanding and avoid mixing up the different concepts. Although there is comparatively little content in this block, it is worth investing time to build confidence with shapes and lines in different orientations. Students can revisit and enhance their knowledge of special triangles and quadrilaterals and focus on key vocabulary such as object, image, congruent etc.
Rotation and translations will be explored in Year 9
National Curriculum content covered includes:

- describe, sketch and draw using conventional terms and notations: points, lines, parallel lines, perpendicular lines, right angles, regular polygons, and other polygons that are reflectively and rotationally symmetric
- identify properties of, and describe the results of reflections applied to given figures

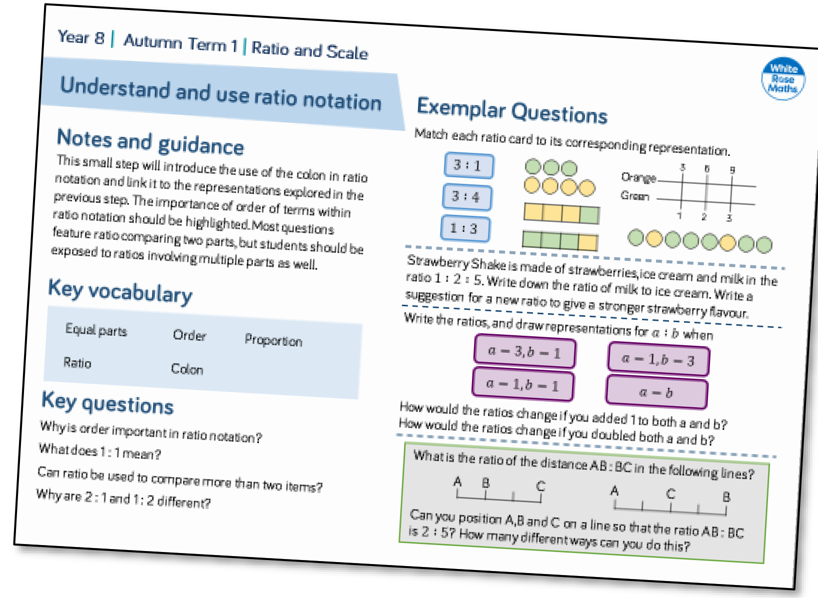
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.



- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Angles in parallel lines & polygons

Small Steps

- Understand and use basic angles rules and notation R
- Investigate angles between parallel lines and the transversal
- Identify and calculate with alternate and corresponding angles
- Identify and calculate with co-interior, alternate and corresponding angles
- Solve complex problems with parallel line angles
- Constructions triangles and special quadrilaterals R
- Investigate the properties of special quadrilaterals
- Identify and calculate with sides and angles in special quadrilaterals

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes 'review step' – content should have been covered in Year 7

White Rose Maths

Angles in parallel lines & polygons

Small Steps

- **Understand and use the properties of diagonals of quadrilaterals** H
- Understand and use the sum of exterior angles of any polygon
- Calculate and use the sum of the interior angles in any polygon
- Calculate missing interior angles in regular polygons
- **Prove simple geometric facts** H
- **Construct an angle bisector** H
- **Construct a perpendicular bisector of a line segment** H

H denotes higher strand and not necessarily content for Higher Tier GCSE

R denotes 'review step' – content should have been covered in Year 7

Basic angle rules and notation R

Notes and guidance

This step revisits key angle facts learnt in Year 7, and reminds students of three-letter angle notation. Students often find this notation difficult, so plenty of practice is helpful here. When finding missing angles, students should justify their answers using fully correct mathematical reasons e.g. "the angles on a straight line add up to 180°," rather than, "it's a straight line".

Key vocabulary

Adjacent	Angles at a point	Vertically Opposite
Straight	Acute/Obtuse/Reflex/Right angle	

Key questions

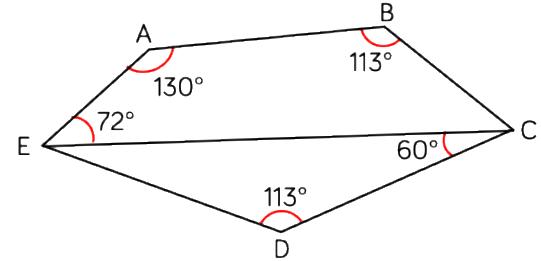
How is a right angle shown on diagrams?

How do you draw an angle of 180°?

What's the difference between an acute angle and an obtuse angle?

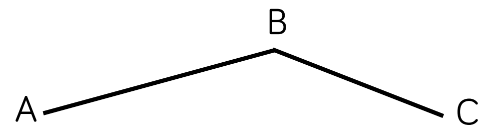
What angle rules do you know? How could they be applied to this diagram?

Exemplar Questions



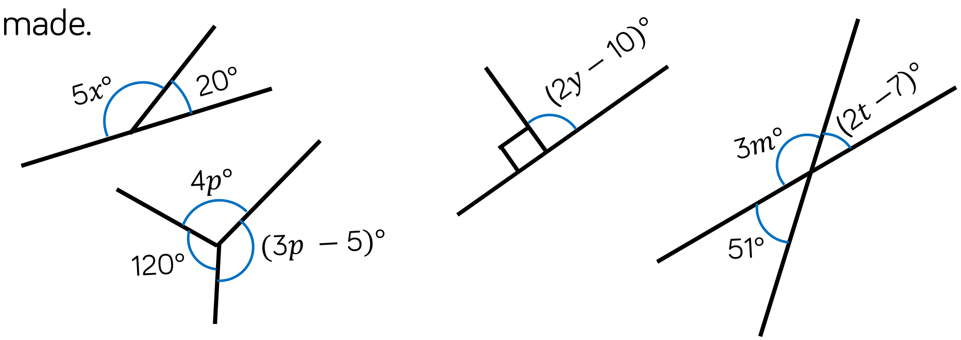
- Angle ABC is the same size as angle ______.
- Angle _____ is 70° greater than angle ______.
- Angle _____ and _____ are examples of acute angles.
- Angle ECB = __________°

Amir says that angle ABC is a reflex angle.



Is Amir correct? How could you be sure?

Form and solve equations to find the values of the unknown letters. Give reasons for your answers and state any assumptions you have made.



Angles between parallel lines

Notes and guidance

In this step, students will investigate angles between parallel lines using vocabulary such as alternate angles, corresponding angles and transversal. It is helpful to include examples and non-examples of parallel lines and find where the relationships hold. Parallel lines should be varied to include horizontal and vertical sets (including more than two lines) as well as other orientations. Dynamic geometry software is extremely useful!

Key vocabulary

Parallel	Transversal	Alternate
Corresponding	Vertically Opposite	

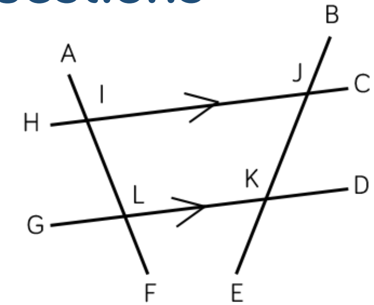
Key questions

How do you know when two or more lines are parallel?

Name a pair of alternate/corresponding angles on the diagram. Which line(s) is/are transversal?

What relationships can you see between the angles? Will this work if you move the transversal line?

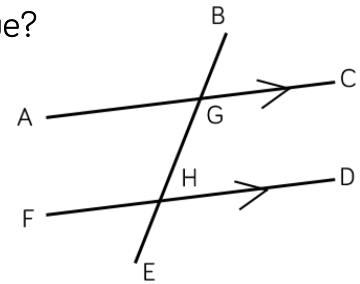
Exemplar Questions

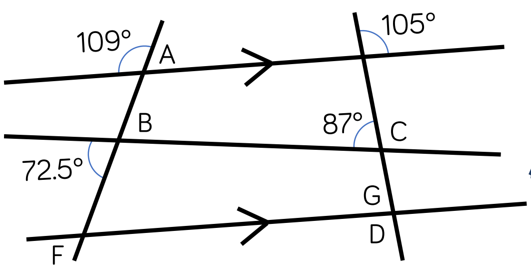


- Lines _____ and _____ are transversal lines.
- Angle GLF is vertically opposite angle _____.
- Measure angle CJK and DKE. What do you notice?
- Measure and label all angles on the diagram.
- Lines HC and GD are parallel because they do not_________.

Which of the following statements are true?

- Lines AC and FD are parallel.
- Angle BGC is equal to BGA.
- Line BE is a transversal line.
- There are four pairs of equal angles.





- Which pairs of angles are alternate and/or corresponding on the diagram?
- When are they equal or not equal?

Alternate & corresponding angles

Notes and guidance

Students will now look more formally at calculating alternate and corresponding angles between parallel lines. Students should also know how to recognise that a pair of lines are parallel because corresponding or alternate angles are equal. As with all angles rules, correct language needs emphasising e.g. "because alternate angles are equal," rather than, "because they are alternate".

Key vocabulary

Angle	Parallel	Transversal
Line	Supplementary	Points

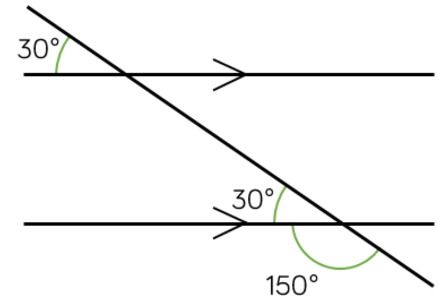
Key questions

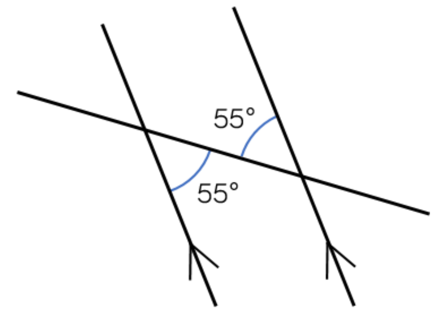
How do you identify a pair of corresponding angles or a pair of alternate angles?

Which angle(s) can you work out directly from the information given on the diagram? What other angle(s) can you then work out?

Exemplar Questions

Work out all of the missing angles. Give reasons for your answers.

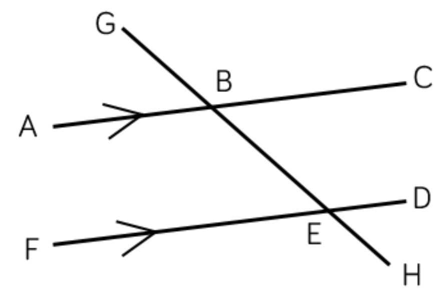




$\angle DEH = 53°$

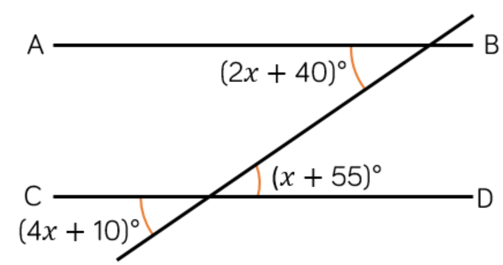
- How can you use this fact to work out $\angle GBC$?
- What other angle(s) do you need to work out first?
- What other angles can you work out?

  Justify your answers.



Form and solve an equation to work out the value of x.

Are line segments AB and CD parallel?



Calculating with co-interior angles

Notes and guidance

Once familiar with working with both corresponding and alternate angles, students can then move on to calculate with co-interior (also known as "allied") angles. Again it is useful to explore examples and non-examples using parallel lines and non-parallel lines to establish whether a given pair will add to give 180 degrees.

Key vocabulary

Parallel	Transversal	Co-interior
Alternate	Corresponding	

Key questions

Why are co-interior angles different to corresponding and alternate angles?

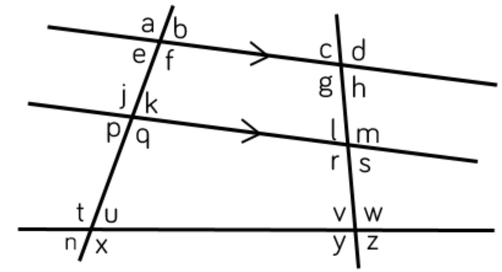
Explain, using understanding of alternate/corresponding angles, why the sum of co-interior angles equal 180°

Can you have co-interior angles in a pair of lines which are not parallel?

Exemplar Questions

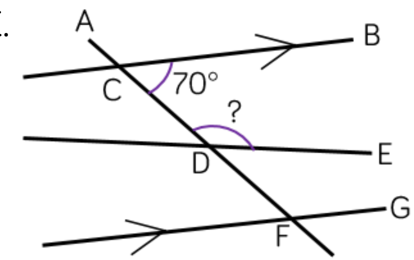
Which pairs of angles sum to 180°?

Which pairs of angles are co-interior?



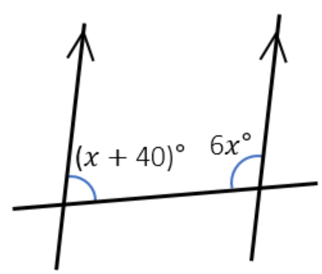
Amir is working out the size of angle ADE.

Angle ADE is 110° because angle BCD is co-interior with angle ADE.



Rosie says that Amir is wrong as there is not enough information. Who do you agree with? Explain why.

Which equations represent the information on the diagram?



$x + 40 = 180$

$x + 40 = 180 - 6x$

$x + 40 + 6x = 180$

$140 - x = 6x$

Work out the size of the labelled angles.

Parallel lines problems

Notes and guidance

In this step, students are exposed to all variations of the angle facts they have learned in the previous steps, including those from previous years. This is an excellent opportunity to develop mathematical talk around the problems, and scaffold their approach through careful questioning. Misconceptions could also be drawn out through 'spot the mistake' examples.

Key vocabulary

Parallel	Transversal	Co-interior
Alternate	Corresponding	

Key questions

What other information do we know that we can add to the diagram?

What tells us if the lines are parallel?

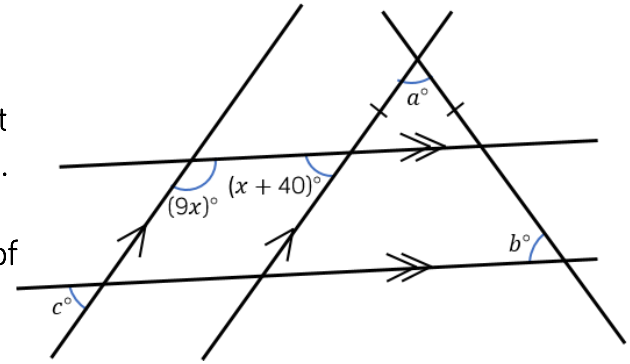
What angle facts do we need to use for this question?

Exemplar Questions

Work out the value of x.

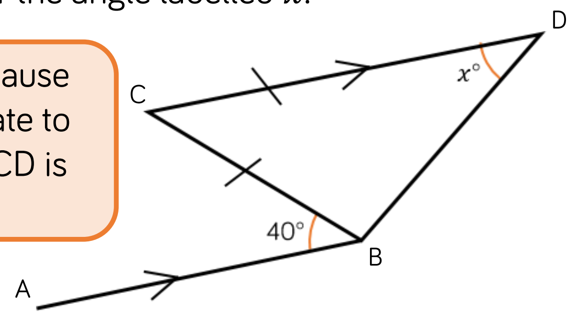
Use your answer to work out the size of angles a, b and c.

Give reasons for each step of your working.



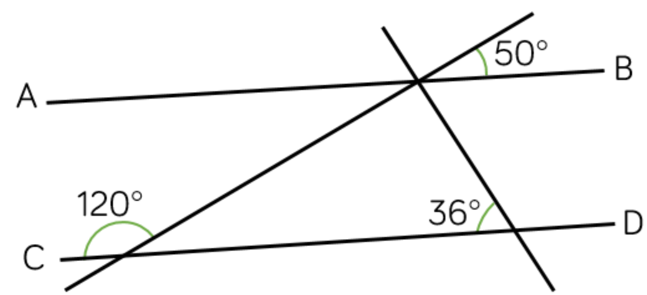
Rosie is calculating the value of the angle labelled x.

x is equal to 40 because angle BCD is alternate to ABC and triangle BCD is isosceles.



What mistake has she made?

Is AB parallel to CD?
Use the given angles to justify your answer.



Triangles and quadrilaterals R

Notes and guidance

In this step, students will revisit constructing triangles given SSS, SAS or ASA, and special quadrilaterals. Students should consider the information given and what mathematical equipment is needed. They can also practise measuring angles by checking each other's work.

Key vocabulary

Isosceles	Equilateral	Scalene
Right-angled	Rhombus	Parallelogram

Key questions

Why don't you need a protractor to draw an equilateral triangle?

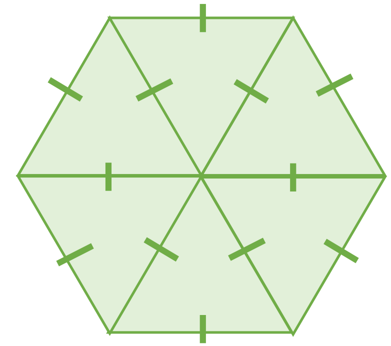
How much information do you need to draw an isosceles triangle?

How is a rhombus different from a parallelogram?

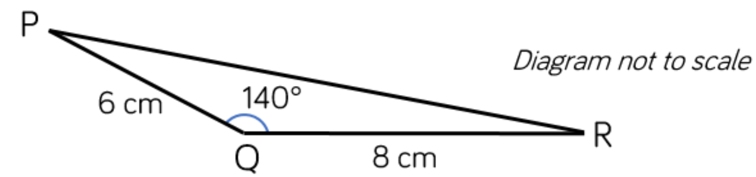
Exemplar Questions

Construct a hexagon of side length 5 cm using only a ruler, a pair of compasses and a pencil.

Use the diagram to help you.



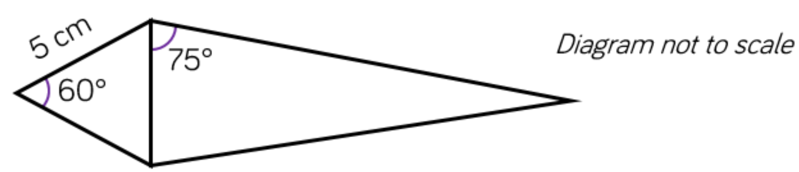
Construct triangle PQR using the information on the diagram.



How may different parallelograms can you make with P, Q and R as three of the vertices?

Construct the quadrilaterals:

- a square of side length 12 cm
- a rhombus ABCD with side lengths 10 cm and ∠ABC = 50°
- a kite as shown in the diagram



Properties of quadrilaterals

Notes and guidance

In this step, students will focus on investigating special quadrilaterals such as squares, rectangles, trapeziums, rhombi and parallelograms. Symmetry will be covered in a later unit, so students should focus on side lengths and angles only. Again, the use of dynamic geometry can help bring the properties to life.

Key vocabulary

Square	Rhombus	Parallelogram
Trapezium	Rectangle	Kite

Key questions

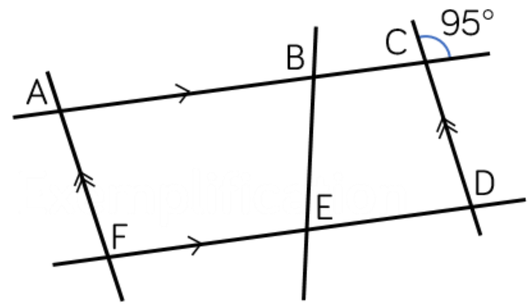
I am a four-sided shape with two pairs of parallel lines, what might I be?

Draw a standard example and a peculiar example of a quadrilateral. Compare your shapes with a partner's.

Which quadrilaterals are regular and which are not?

Exemplar Questions

AF is parallel to CD and AC is parallel to FD.



Explain whether the following statements are true or false.

- ACDF is a rectangle
- ABEF and BCDE are trapezia
- CDFA is a parallelogram

Which quadrilaterals satisfy each of these statements?

Two pairs of parallel sides	Only one pair of parallel sides
Four right angles	Two pairs of equal sides
Two pairs of equal angles	One pair of equal angles

How many quadrilaterals satisfy 2, 3 or more of the statements.

Which of this statements are true and which are false?

- All squares are rectangles
- All rectangles are squares
- No rhombuses are kites

Calculate sides and angles

Notes and guidance

This step could be taught in conjunction with the previous step, with students focusing on applying their knowledge of angle facts and properties of parallel lines to investigate special quadrilaterals and deduce unknown information. Students should be encouraged to discuss and label what information they know or can work out on their diagrams.

Key vocabulary

Rhombus	Parallelogram	Trapezium	Kite
Rectangle	Isosceles	Equilateral	

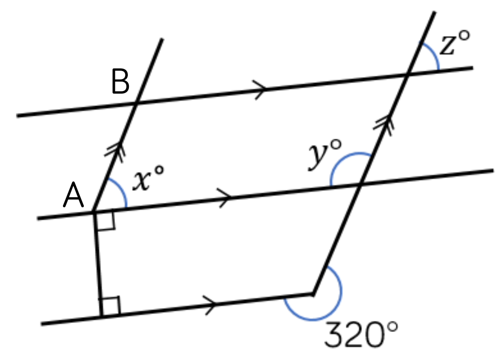
Key questions

What properties does a rhombus have that a parallelogram does not? What similar properties do they have?

Give me an example of a quadrilateral which only has one obtuse angle/two obtuse angles.

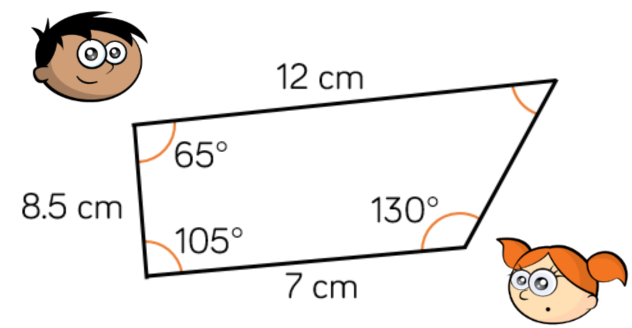
What makes a trapezium an isosceles trapezium?

Exemplar Questions

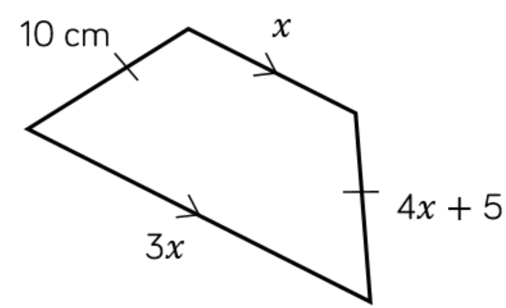


Using angle facts, find the unknown angles x, y and z.
What quadrilaterals can you see in the diagram?
How did you decide?
If you knew the length of AB, what else could you work out?

Amir says the shape is a trapezium. Rosie thinks there's not enough information to be able to tell.
Who do you agree with? Why?



Work out the perimeter of the isosceles trapezium.



Diagonals of quadrilaterals H

Notes and guidance

Students should be aware that a diagonal joins the opposite vertices of a quadrilateral, and that they don't necessarily "go diagonally"! Geoboards and/or squared paper are very useful for discovering properties, as is dynamic geometry. It is also useful to use straws of various sizes to reverse the process, starting with diagonals with certain properties and deducing what the quadrilateral must be.

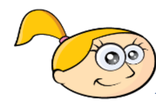
Key vocabulary

Rhombus	Parallelogram	Trapezium	Kite
Rectangle	Perpendicular	Bisect	Delta

Key questions

Is it possible for the diagonals of a quadrilateral to be horizontal or vertical?
What types of quadrilateral have diagonals that are equal in length? Why can't this be the case for the other special quadrilaterals?
Is it possible for a diagonal to be outside the shape?

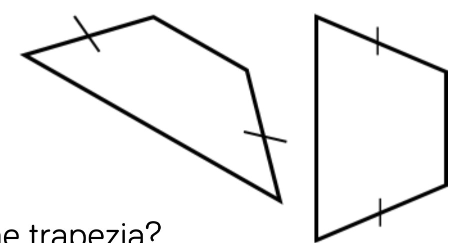
Exemplar Questions



I'm thinking of a quadrilateral.
The diagonals of my quadrilateral are perpendicular.

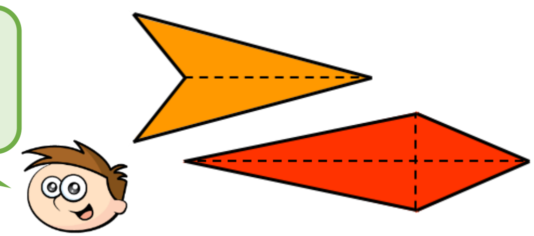
- What are the three types of quadrilateral Eva could be thinking of?
- Is it possible for the diagonals of these quadrilaterals to also be equal in length?
- Is it possible for the diagonals of these quadrilaterals to also bisect each other?

Draw some different isosceles trapezia and investigate their diagonals.



- Do they bisect each other?
- Do they bisect the angles of the trapezia?
- Is it possible for them to be equal in length?
  Use dynamic geometry to check your findings.

A delta only has one diagonal, but a kite has two.



Do you agree with Teddy?

Sum of exterior angles

Notes and guidance

In this step, students should explore the meaning of external angles and how to find them by extending the lines of a polygon. Using a pen or pencil to go around the outside of a polygon from one side to the next demonstrates that there is one full turn needed, no matter how many sides, and so the exterior angle sum is always 360°. Students need to know that the exterior angles will only be equal if the polygon is regular.

Key vocabulary

Exterior	Interior	Regular	Polygon
Sum	Total	Pentagon/Hexagon etc.	

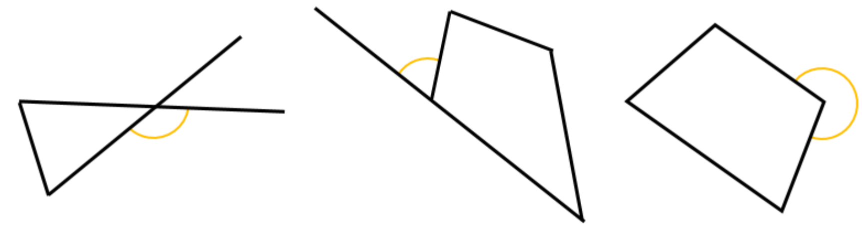
Key questions

What are the two conditions that make a polygon regular?

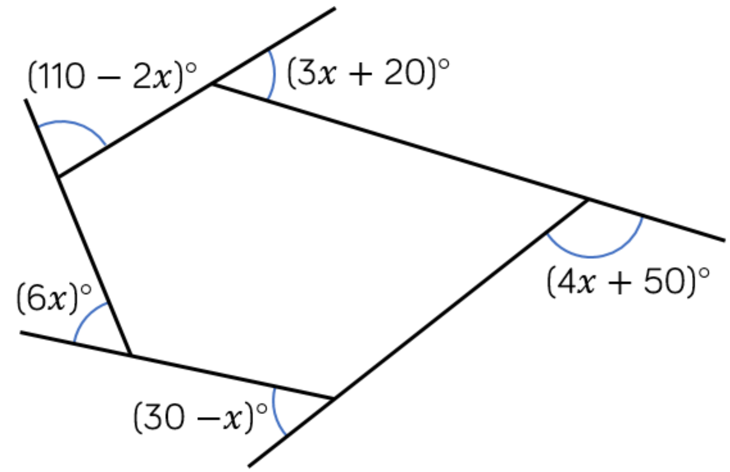
What is the sum of the external angles of a polygon? If the polygon is regular, what is the size of each external angle?

Exemplar Questions

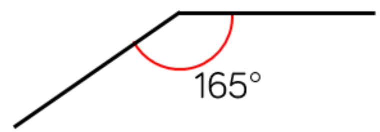
Which of the following diagrams shows an exterior angle of the shape?



Form and solve an equation to find the size of the largest external angle of the pentagon.



The diagram shows 2 sides of a regular polygon.
How many sides does the polygon have?



Sum of interior angles in polygons

Notes and guidance

In this step, students should explore the sum of the interior angles in different polygons – students following the Higher strand may have covered this last year. Students should explore the links between the number of sides a polygon has and the number of internal triangles a polygon has, and so deduce the interior angle sum is given by $(n-2) \times 180°$ They explore a regular polygon's angles in the next step.

Key vocabulary

Exterior	Interior	Regular	Polygon
Sum	Total	Pentagon/Hexagon etc.	

Key questions

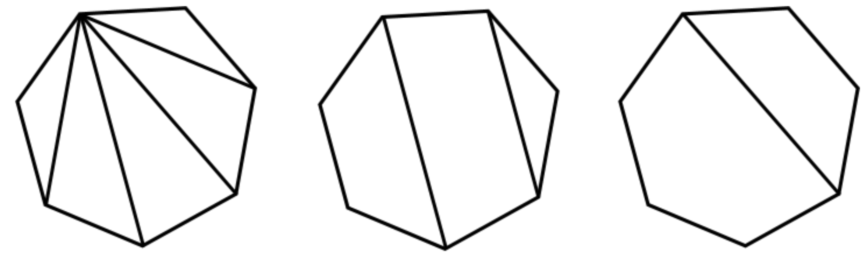
If a polygon is regular, what do we know about its angles?

Will the interior angles of a 20-sided shape be greater than or less than those of a 19-sided shape? What about the exterior angles?

Is it possible to have a reflex interior angle in a polygon? Give me an example.

Exemplar Questions

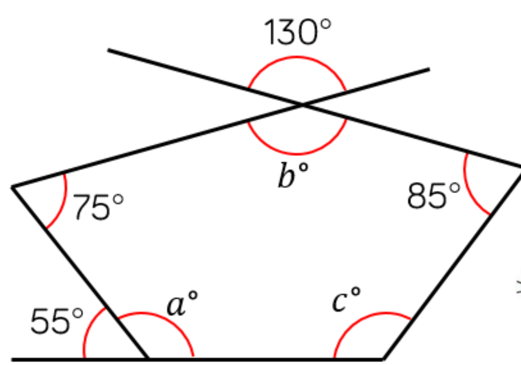
Which of the following diagrams would be helpful in finding the sum of the interior angles of regular heptagon?



Annie is calculating the sum of interior angles in a 12-sided shape.

The angles in a quadrilateral sum to 360°
A 12-sided shape has 3 times as many sides, so its angle sum will be $3 \times 360° = 1080°$

Prove that Annie is wrong.



Calculate the unknown angles in this polygon.
Give mathematical reasons for all your answers.

Does the order in which you find the angles matter?

Missing angles in regular polygons

Notes and guidance

Students sometimes misunderstand 'regular' as only meaning equal sides, or even rectilinear, so this is a good opportunity to discuss regularity whilst using the recently learnt rules of interior and exterior angle sums. It is also useful to compare different methods to find the size of one interior angle. Students could take this further, exploring which regular polygons tessellate and why.

Key vocabulary

Exterior	Interior	Regular	Polygon
Sum	Total	Pentagon/Hexagon etc.	

Key questions

Will the interior angles of a regular polygon be different from those of an irregular polygon?

Explain why neither a rectangle nor a rhombus are regular.

What's the connection between the interior and the exterior angles of a polygon?

Exemplar Questions

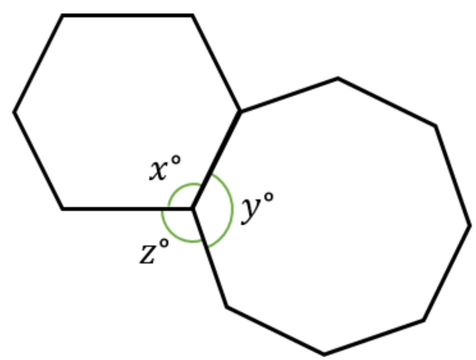
Which of the following calculations would be correct for working out the size of one interior angle of a regular decagon?

$10 - 2 = 8$ $360° \div 8 = 45°$	$10 - 2 = 8$ $8 \times 180° = 1440°$ $1440 \div 10 = 144°$	$10 + 2 = 12$ $12 \times 180° = 2160°$ $2160 \div 10 = 216°$

Compare Whitney and Tommy's methods for working out the size of an interior angle of a regular nonagon.

The interior angle sum is
$(9 - 2) \times 180° = 1260°$
So one angle is
$1260° \div 9 = 140°$

Each exterior angle is $360° \div 9 = 40°$
So each interior angle is $180 - 40° = 140°$



The diagram shows a regular hexagon and a regular octagon that meet at a common point.

Work out the values of x, y and z.

Prove geometric facts

H

Notes and guidance

The formal proof that the sum of the angles in a triangle is 180° is a good introduction to this step as it builds on previous work. It is also useful to compare this to tearing corners off a triangle, illustrating the difference between proof and demonstration. Students could then do small "show that" activities (e.g. the value of a missing angle) before moving on to short formal proofs like those illustrated in the exemplars.

Key vocabulary

Demonstration Justify Proof

Alternate/Corresponding/Parallel etc.

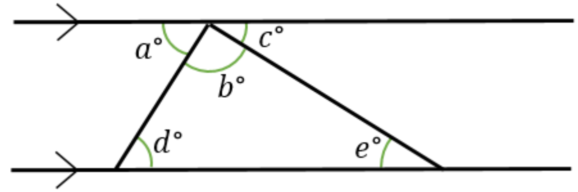
Key questions

What's the difference between a proof and a demonstration?

How do we know the result will always be true?

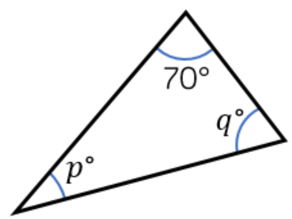
What can we find out first?

Exemplar Questions

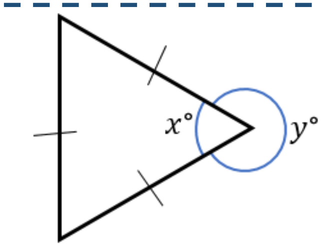


Complete the proof that angles in a triangle add to 180°

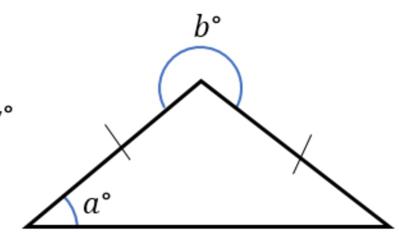
$a + b + c = 180°$ (Angles on a ________add up to 180°)
$a = d$ (Alternate angles are _____)
$c =$ __ (_____ angles are _____)
So $a + b + c = d + b +$ __
So $d + b + e = 180°$



Prove
$p + q = 110°$

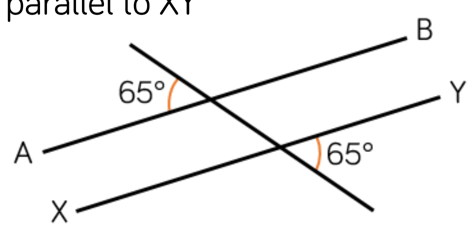


Show that
$y = 300°$



Prove
$b = 180 + 2a$

Prove that AB is parallel to XY



Angle bisectors

H

Notes and guidance

It is useful to use a visualiser to demonstrate the method of bisecting an angle using only a pencil and a pair of compasses. Students should practise with angles of different sizes and in different orientations. This topic does not link easily to other areas of maths, so it might be worthwhile revisiting occasionally as a starter (comparing with the next step) to help students to remember the technique.

Key vocabulary

Bisect	Bisector	Acute
Obtuse	Reflex	Compasses

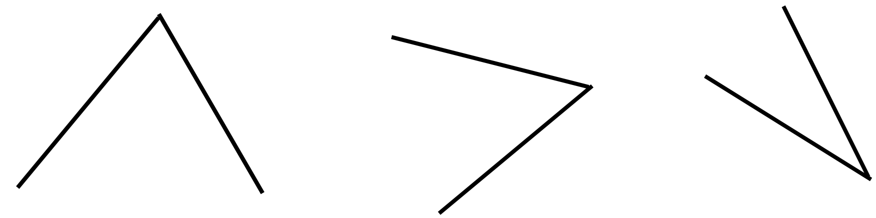
Key questions

What does bisect mean? What does the stem "bi" tell us?

Describe the steps to construct the bisector of an angle without using a protractor.

Exemplar Questions

Bisect the acute angles.

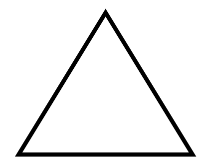


Use a protractor to draw angles of these sizes.

- 140°
- 170°
- 90°
- 10°

Now use a pair of compasses to help bisect the angles, and check how accurate you were with a protractor.

Thinking about how you would construct an equilateral triangle, construct an angle of 60° without using a protractor.
Now construct the angles on the cards.



30° 120° 150° 210°

Draw a large acute-angled triangle.
Bisect each angle.
Using the point where the three angle bisectors meet as the centre, draw a circle that just touches each of the three sides of the triangle.
(This is called the 'incircle' of the triangle).

Perpendicular line bisectors H

Notes and guidance

As with angle bisectors, it is useful to use a visualiser to demonstrate the method of bisecting an angle using only a pencil and a pair of compasses. Students should again practise with lines of different size and in different orientations. This topic does not link easily to other areas of maths, so it might be worthwhile revisiting occasionally as a starter (comparing with the last step) to help students to remember the technique.

Key vocabulary

Line	Line segment
Perpendicular	Bisector

Key questions

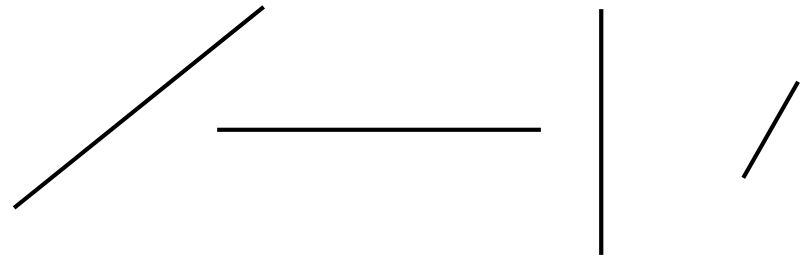
Tell me what perpendicular means?

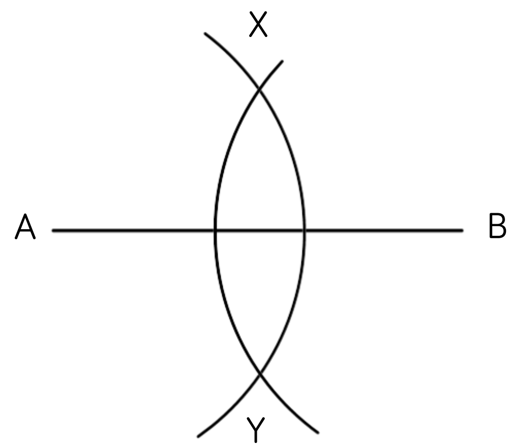
What does bisect mean? What does the stem "bi" tell us?

What's the connection between the method for constructing a perpendicular bisector and what we know about the diagonals of a rhombus?

Exemplar Questions

Construct the perpendicular bisectors of these line segments.





Construct the perpendicular bisector, XY, of a line segment AB as shown, leaving on all the construction lines.
Join the points to form the quadrilateral AXBY.

- What type of quadrilateral is AXBY?
- What so you notice about its diagonals?

Draw a large acute-angled triangle.
Bisect each side.
Using the point where the three angle bisectors meet as the centre, draw a circle that just touches each of the vertices of the triangle.
(This is called the 'circumcircle' of the triangle)