Multiplicative Change

Year 8

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability
Spring	**Algebraic techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages			Standard index form		Number sense
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons			Area of trapezia and circles		Line symmetry and reflection	The data handling cycle				Measures of location	

Autumn 1: Proportional Reasoning

Weeks 1 and 2: Ratio and Scale

This unit focuses initially on the meaning of ratio and the various models that can be used to represent ratios. Based on this understanding, it moves on to sharing in a ratio given the whole or one of the parts, and how to use e.g. bar models to ensure the correct approach to solving a problem. After this we look at simplifying ratios, using previous answers to deepen the understanding of equivalent ratio rather than 'cancelling' purely as a procedure. We also explore the links between ratio and fractions and understand and use π as the ratio of the circumference of a circle to its diameter. Students following the higher strand also look at gradient in preparation for next half term.

National Curriculum content covered includes:

- make connections between number relationships, and their algebraic and graphical representations
- use scale factors, scale diagrams and maps
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio
- solve problems involving direct and inverse proportion

Weeks 3 and 4: Multiplicative Change

Students now work with the link between ratio and scaling, including the idea of direct proportion, linking various form including graphs and using context such as conversion of currencies which provides rich opportunities for problem solving. Conversion graphs will be looked at in this block and could be revisited in the more formal graphical work later in the term. Links are also made with maps and scales, and with the use of scale factors to find missing lengths in pairs of similar shapes.

National Curriculum content covered includes:

- extend and formalise their knowledge of ratio and proportion in working with measures and in formulating proportional relations algebraically
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- use scale factors, scale diagrams and maps
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- move freely between different numerical, algebraic, graphical and diagrammatic representations

Weeks 5 and 6: Multiplying and Dividing Fractions

Students will have had a little experience of multiplying and dividing fractions in Year 6; here we seek to deepen understanding by looking at multiple representations to see what underpins the (often confusing) algorithms. Multiplication and division by both integers and fractions are covered, with an emphasis on the understanding of the reciprocal and its uses. Links between fractions and decimals are also revisited. Students following the Higher strand will also cover multiplying and diving with mixed numbers and improper fractions.

National Curriculum content covered includes:

- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals and fractions
- select and use appropriate calculation strategies to solve increasingly complex problems
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative

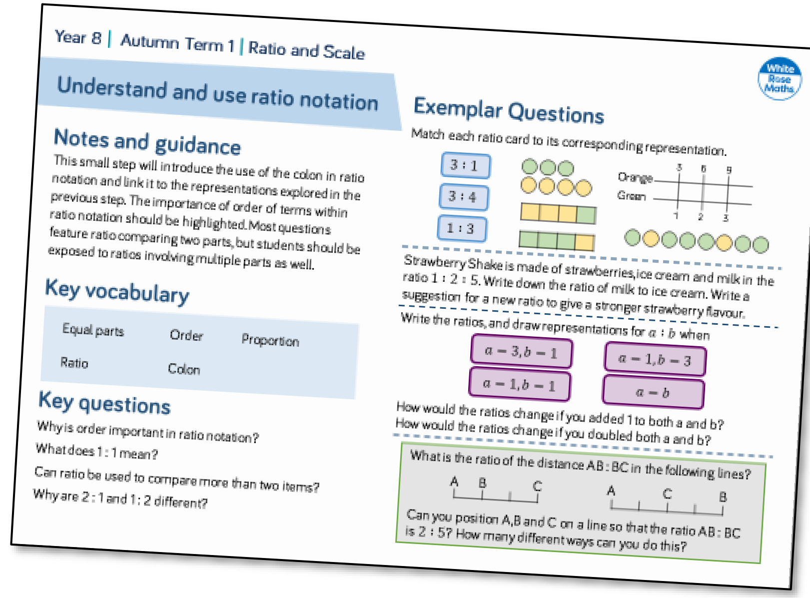
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a “small steps” approach.

As a result, for each block of content in the scheme of learning we will provide a “small step” breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students’ attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.


Year 8 | Autumn Term 1 | Ratio and Scale

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts, Order, Proportion, Ratio, Colon

Key questions

Why is order important in ratio notation?
What does 1 : 1 mean?
Can ratio be used to compare more than two items?
Why are 2 : 1 and 1 : 2 different?

Exemplar Questions

Match each ratio card to its corresponding representation.

3 : 1, 3 : 4, 1 : 3

Strawberry Shake is made of strawberries, ice cream and milk in the ratio 1 : 2 : 5. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.

Write the ratios, and draw representations for $a : b$ when

$a = 3, b = 1$; $a = 1, b = 3$; $a = 1, b = 1$; $a = b$

How would the ratios change if you added 1 to both a and b?
How would the ratios change if you doubled both a and b?

What is the ratio of the distance AB : BC in the following lines?

Can you position A, B and C on a line so that the ratio AB : BC is 2 : 5? How many different ways can you do this?

- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

Multiplicative Change

Small Steps

- Solve problems involving direct proportion
- Explore conversion graphs
- Convert between currencies
- Explore direct proportion graphs H
- Explore relationships between similar shapes
- Understand scale factors as multiplicative representations
- Draw and interpret scale diagrams
- Interpret maps using scale factors and ratios

H denotes higher strand and not necessarily content for Higher Tier GCSE

Direct proportion

Notes and guidance

In this small step we will explore the fundamentals of direct proportion. Students could think of examples of direct proportion in real life, where as one variable doubles, so does the other. Multiple methods should be explored to give students strategies for the variety of problems that can be posed. Some of these methods are shown in the exemplar questions.

Key vocabulary

Proportion	Ratio	Double
Triple	Linear	Variable

Key questions

How is direct proportion similar to times tables?

If two variable quantities are in direct proportion, what happens if you halve the value of one variable?

What happens if you triple the value of one variable?

Is direct proportion linked to ratio?

Exemplar Questions

5 scoops of ice cream costs £4.50. How much would it cost for:

- 10 scoops
- 8 scoops
- 1 scoop
- 9 scoops

Which of these situations are direct proportion?

'If you double the number of cans, you double the cost'

'Every time you add an extra can the total weight increases by the same'

'If you have 20% fewer cans, you will have 20% less volume'

'If there are 4 times as many people, we will need 4 times as many cans'

'Every extra row of pop is an extra 4 cans'

'If you have no cans, it will cost nothing'

The recipe has been stained. Use everyone's working out to find the missing information.

Carina is making 50 muffins.
50 = '2 and a half lots of 20'
2.5 × 250 = 625 g of sugar

Zaib is making 12 muffins
20 : 250 ml
1 : 12.5 ml
12 : 150 ml
150 ml of milk

Emma is making 80 muffins.

20	20	20	20

8 eggs

Daniel is making 5 muffins.
20 ÷ 5 = **4**
"I need **4** times less than the recipe I will use 100g of flour".

White Rose Maths

Explore conversion graphs

Notes and guidance

This is a particular skill that students will come across in many other topics in school.

For more precise conversions, graph paper should be used practising the use of scales. Students should be encouraged to draw vertical and horizontal lines for the most accurate conversions from their graphs.

Key vocabulary

Linear	Axes	Labelling
Units	Conversion	Approximation

Key questions

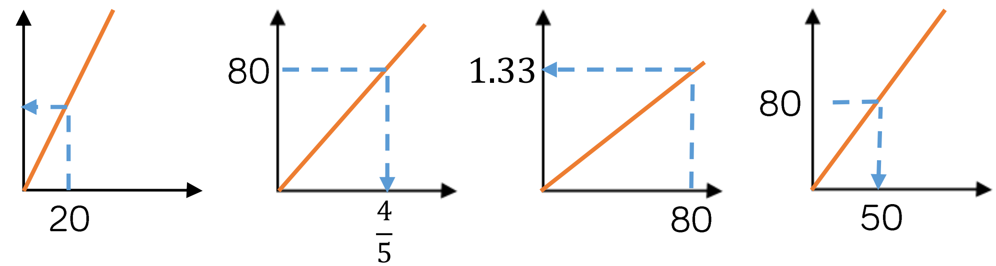
Do all conversion graphs start at the origin?

Is it important to label axes on a conversion graph?

What should the limits of your axes be?

Exemplar Questions

Match the conversion graph to the statement.



Convert width of a square to perimeter

Convert kilometres to miles

Convert centimetres to metres

Convert seconds to minutes

Draw an accurate version of each graph, labelling your axes. Think carefully about the choice of scales.

Draw a conversion graph for converting Fahrenheit to Celsius given the following facts:

- Water freezes at 32 degrees Fahrenheit (32°F)
- Water boils at 212 degrees Fahrenheit
- −40 is the same in Celsius and Fahrenheit

Amir is asked to convert decimals to percentage.
He decides to draw a graph.
What are the features of his graph?
Can he use it to convert fractions to decimals?

Convert between currencies

Notes and guidance

Conversion of currency brings together many of the ideas covered in previous small steps. It is useful to explore the many different methods that can be employed in this topic. Students should be encouraged to estimate their answers before converting to ensure they have a sensible answer. Both calculator and non-calculator should be explored, considering which is appropriate when.

Key vocabulary

Exchange rate	Currency	Conversion
Estimate	Sterling	

Key questions

How is the conversion of pounds to dollars different to dollars to pounds?

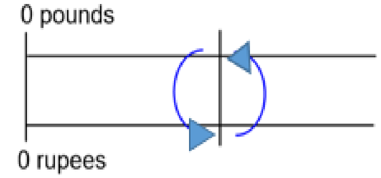
How do conversion rates relate to ratios?

Is converting a currency an example of direct proportion?

Exemplar Questions

£1 = 90 Rupees

Copy and complete the number line.



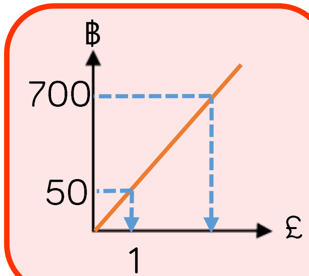
Alex calculates changing 200 rupees into
Her answer is £18 000
Does this seem right?
Explain why or why not.

Write a sentence explaining what each of these calculations works out.

400×90 | $400 \div 90$ | 800×90

---

1 British pound (£) is approximately 50 Thai Baht (฿)
Explain how each of these representations could be used to convert 700฿ into pounds. Why do they all work?



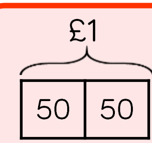
$\times 0.02$

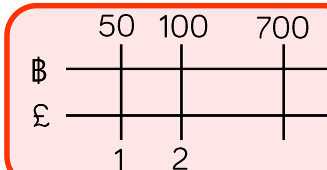
฿ → $\div 100$ → $\times 2$ → £

$700฿ \times \left(\frac{£1}{100฿}\right)$

$p = \frac{b}{50}$
where p= number of pounds and b= number of baht

$1 : 50$
$? : 700$





Direct proportion graphs

H

Notes and guidance

Encourage students to think of where they might find direct proportion in real life and to represent their ideas as graphs. Comparing these will help to cement some of the main features of the graphs. Remind students of the importance of labelling their axes and finding an appropriate scale. Students can be asked questions involving values outside those in the conversion graph.

Key vocabulary

Rate	Directly proportional	Origin
Constant	Relationship	Linear

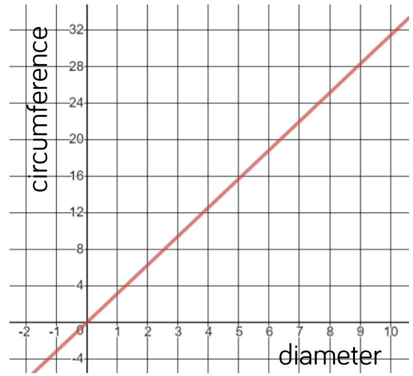
Key questions

Do all direct proportion graphs start at the origin?

How might we use the graph to answer questions that use values beyond those on the axis?

Why is it important to label the axes?

Exemplar Questions



The circumference of a circle is directly proportional to its diameter.

Use the graph to estimate the circumference of a circle with diameter:

- 4 m
- 10 m
- 14 m
- 140 cm
- 0.14 inches

Are these statements true or false?

The graph shows a constant rate of increase.

A circle with a negative diameter has a negative circumference.

All direct proportion graphs go though the origin.

5 scoops of ice cream cost £4.50. Draw a graph that will show the price of up to 40 scoops of ice cream.
How could you use this to find the price difference between 15 and 24 scoops?

If a car is travelling at a *constant* speed, the distance it travels is directly proportional to the time it has been travelling.
Complete the table and draw the graph, describing its key features.

Time (mins)	30	60		114.2
Distance (miles)	18		300	

Ratio between similar shapes

Notes and guidance

Students have already been briefly exposed to similar shapes in their exploration of π and gradient. In this small step we will focus on the fact that corresponding lengths on similar shapes are in the same ratio. Students should be familiar with similar shapes presented in different orientations. Exploration of examples and non-examples is useful here to cement the concept of similar shapes.

Key vocabulary

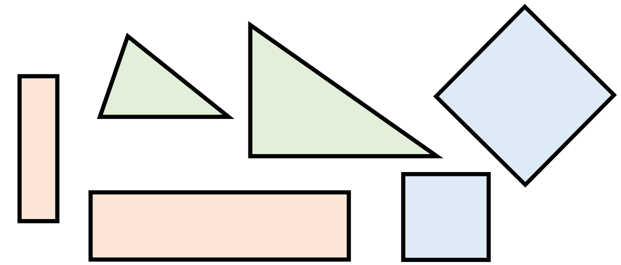
Orientation	Similar	Corresponding
Proportion		

Key questions

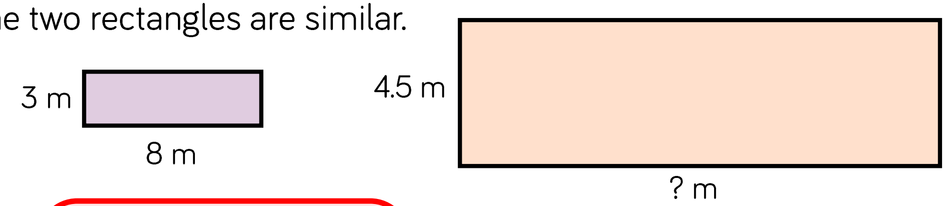
What do you notice about the angles in a pair of similar shapes?
If shapes are not drawn to scale, how can we show they are similar?
How can labelling the vertices be useful with similar shapes?

Exemplar Questions

These shapes are drawn to scale.
Which pairs of shapes are similar?
How can you be sure that they are similar?



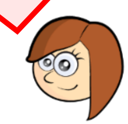
The two rectangles are similar.



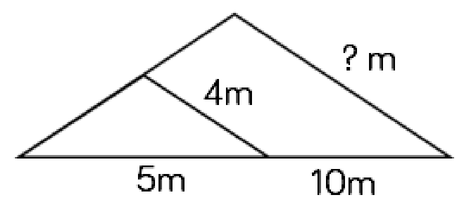
The height has gone up by 1.5 m, so the width of the orange rectangle is 9.5 m.

The ratio of the height of the purple to orange rectangle is 2 : 3

Do you agree with Rosie or Tommy?
Explain your answer.



Two triangles are similar.
Find the missing length.



Understand scale factors

Notes and guidance

Bringing together work on ratio of $1 : n$ and similar shapes, this small step introduces enlargement scale factors, and could be taught in conjunction with the similar shapes small step.

The focus is on length scale factor, not area or volume, though some students may naturally make observations.

Key vocabulary

Scale factor	Enlargement	Object
Image	Length	

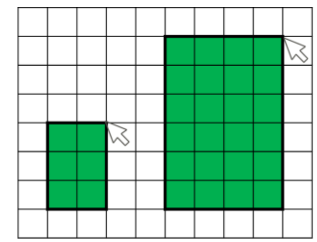
Key questions

How does a scale factor compare to a ratio?
What range of scale factors would make an image smaller?
If the lengths of a shape have tripled, what is the scale factor?

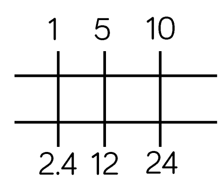
Exemplar Questions

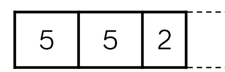
The green rectangle is enlarged on a computer. Which of the statements are true?

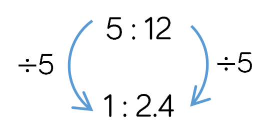
- The height ratio from object to image is 3 : 6
- The width ratio from object to image is 2 : 4
- The ratio of 2 corresponding lengths is 1 : 2
- The image is enlarged by a scale factor of 2
- The image is twice as big as the object.
- The diagonals of the larger rectangle are twice the length of those of the smaller rectangle.

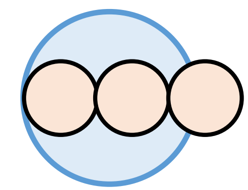


The radius of a circle is enlarged from 5 cm to 12 cm.
How do these images represent the fact that the scale factor is 2.4?









An object is enlarged by a scale factor of 0.5
Which of the following is true? (You could draw a shape and it’s enlargement on squared paper to help you)

- The sides of the new image are half as long.
- An enlargement can make shapes smaller.
- The area of the image is half the area of the object.

Draw and interpret scale diagrams

Notes and guidance

Students should explore this step practically by creating and using scale drawings of items from the classroom etc.
The link between scale, scale factors and ratio needs to be made explicit. This could be reinforced by linking back to earlier representations such as the double number line.
Examples of diagrams that are not to scale could be useful to emphasise the key features of scale.

Key vocabulary

Scale factor	Not to scale	Enlargement
Plan	Image	

Key questions

Are scale diagrams always smaller versions of the original?

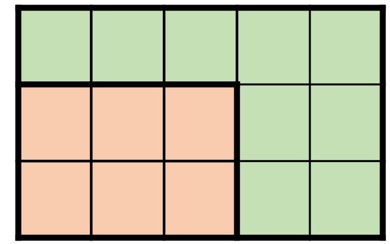
Why is a scale diagram useful?

Describe a method for finding an appropriate scale.

Exemplar Questions

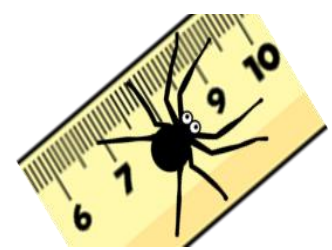
Tommy is planning where to build his house on his land.
He has drawn one idea on some cm squared paper. 1 cm = 3 m

	Tommy's House
	Tommy's Garden



What is the actual width of his plot of land?
What is the actual perimeter of the house?
What is the actual area of the land left after building the house?

A biologist is drawing a diagram of a house-spider to go in a book.
She wants it to fill a whole A4 page.
What scale factor could she use?
How would the scale factor change if she wanted twice as many spiders on a page?



Samira draws a picture of a car with a scale of 1 : 30
The car is 3 m long. How long is her scale drawing?
The height of the car on the picture is 5 cm.
What is the height of the actual car?

White Rose Maths

Interpret maps with scale factors

Notes and guidance

Teachers might consider revisiting metric unit conversions before starting this small step. Specifically, students need to be confident in working with large numbers (for example, above 10 000). This small step could be introduced using real-life maps, and the meaning of each scale. Using representations, such as double number lines, will help students to connect this small step to previous ones.

Key vocabulary

Distance Conversion Units

Metric

Key questions

What does the scale 1 : 25 000 mean on a map?
Can you express it as a ratio in mixed units?

Would a map with scale 1 : 25 000 need to be bigger or smaller than a map with scale of 1 : 1250 showing the same features?

Exemplar Questions

Explain why each of these describes a scale of 1 : 25 000

1 cm on the map is 25 000 cm in real life.

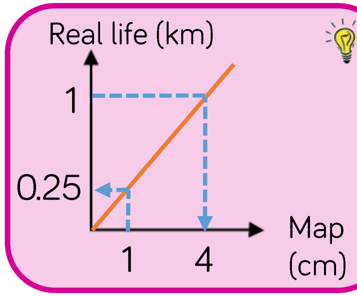
$1 : 2.5 \times 10^4$

1 cm : 25 000 cm

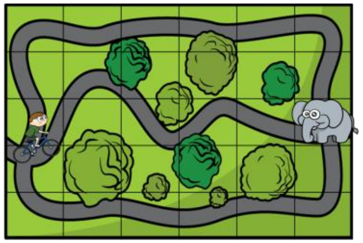
1 cm : 250 m

1 cm : 0.25 km

4 cm on the map is 1 km on the ground.



A pirate sails from her island to find treasure.
She travels 15 km North, turns East and sails 30 km, and then turns North again for the final 40 km to take her to some treasure. Draw a scale map of her journey using a scale of 1 : 500 000
Her parrot flies directly from the island to the treasure.
Use your map to find out how much further the pirate travelled than the parrot.



The scale of this map is 1 : 1250
Each square is 1 cm by 1 cm.
Which is the shortest route for the boy to cycle to his elephant?