Ratio and Scale

Year 8

#MathsEveryoneCan

White Rose Maths

	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12
Autumn	**Proportional Reasoning**						**Representations**					
	Ratio and scale		Multiplicative change		Multiplying and dividing fractions		Working in the Cartesian plane			Representing data		Tables & Probability
Spring	**Algebraic techniques**						**Developing Number**					
	Brackets, equations and inequalities				Sequences	Indices	Fractions and percentages			Standard index form		Number sense
Summer	**Developing Geometry**						**Reasoning with Data**					
	Angles in parallel lines and polygons			Area of trapezia and circles		Line symmetry and reflection	The data handling cycle				Measures of location	

Autumn 1: Proportional Reasoning

Weeks 1 and 2: Ratio and Scale

This unit focuses initially on the meaning of ratio and the various models that can be used to represent ratios. Based on this understanding, it moves on to sharing in a ratio given the whole or one of the parts, and how to use e.g. bar models to ensure the correct approach to solving a problem. After this we look at simplifying ratios, using previous answers to deepen the understanding of equivalent ratio rather than 'cancelling' purely as a procedure. We also explore the links between ratio and fractions and understand and use π as the ratio of the circumference of a circle to its diameter. Students following the higher strand also look at gradient in preparation for next half term.

National Curriculum content covered includes:
- make connections between number relationships, and their algebraic and graphical representations
- use scale factors, scale diagrams and maps
- understand that a multiplicative relationship between two quantities can be expressed as a ratio or a fraction
- divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio
- solve problems involving direct and inverse proportion

Weeks 3 and 4: Multiplicative Change

Students now work with the link between ratio and scaling, including the idea of direct proportion, linking various form including graphs and using context such as conversion of currencies which provides rich opportunities for problem solving. Conversion graphs will be looked at in this block and could be revisited in the more formal graphical work later in the term. Links are also made with maps and scales, and with the use of scale factors to find missing lengths in pairs of similar shapes.

National Curriculum content covered includes:
- extend and formalise their knowledge of ratio and proportion in working with measures and in formulating proportional relations algebraically
- interpret when the structure of a numerical problem requires additive, multiplicative or proportional reasoning
- use scale factors, scale diagrams and maps
- solve problems involving direct and inverse proportion, including graphical and algebraic representations
- move freely between different numerical, algebraic, graphical and diagrammatic representations

Weeks 5 and 6: Multiplying and Dividing Fractions

Students will have had a little experience of multiplying and dividing fractions in Year 6; here we seek to deepen understanding by looking at multiple representations to see what underpins the (often confusing) algorithms. Multiplication and division by both integers and fractions are covered, with an emphasis on the understanding of the reciprocal and its uses. Links between fractions and decimals are also revisited. Students following the Higher strand will also cover multiplying and diving with mixed numbers and improper fractions.

National Curriculum content covered includes:
- consolidate their numerical and mathematical capability from key stage 2 and extend their understanding of the number system and place value to include decimals and fractions
- select and use appropriate calculation strategies to solve increasingly complex problems
- use the four operations, including formal written methods, applied to integers, decimals, proper and improper fractions, and mixed numbers, all both positive and negative

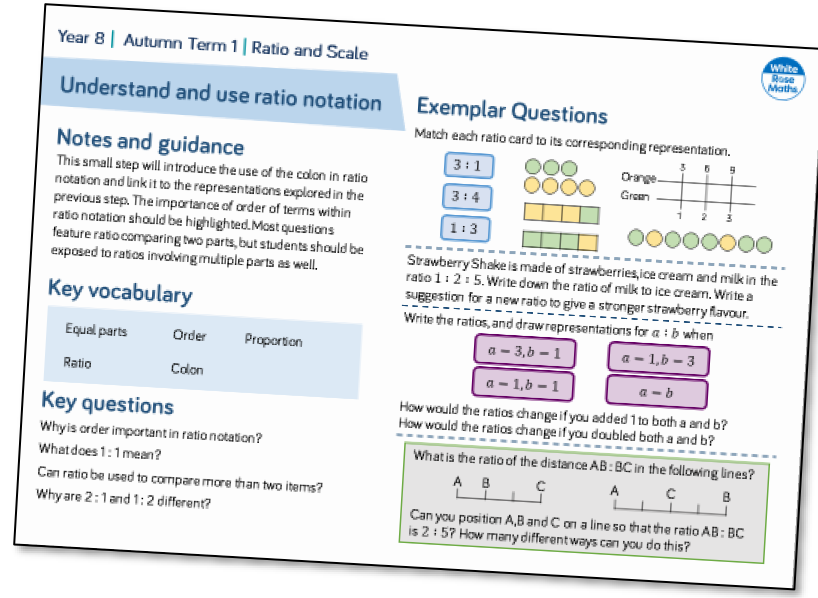
Why Small Steps?

We know that breaking the curriculum down into small manageable steps should help students to understand concepts better. Too often, we have noticed that teachers will try and cover too many concepts at once and this can lead to cognitive overload. We believe it is better to follow a "small steps" approach.

As a result, for each block of content in the scheme of learning we will provide a "small step" breakdown. ***It is not the intention that each small step should last a lesson – some will be a short step within a lesson, some will take longer than a lesson.*** We would encourage teachers to spend the appropriate amount of time on each step for their group, and to teach some of the steps alongside each other if necessary.

What We Provide

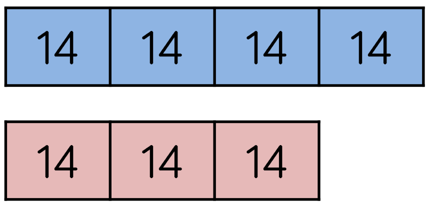
- Some ***brief guidance*** notes to help identify key teaching and learning points
- A list of ***key vocabulary*** that we would expect teachers to draw to students' attention when teaching the small step,
- A series of ***key questions*** to incorporate in lessons to aid mathematical thinking.
- A set of questions to help ***exemplify*** the small step concept that needs to be focussed on.

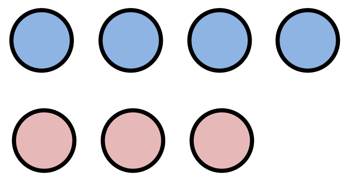


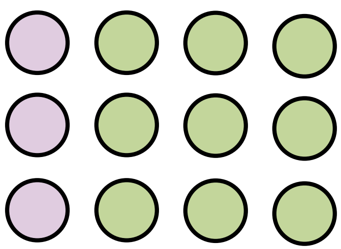
- These include reasoning and problem-solving questions that are fully integrated into the scheme of learning. Depending on the attainment of your students, you many wish to use some or all of these exemplars, which are in approximate order of difficulty. Particularly challenging questions are indicated with the symbol .
- For each block, we also provide ideas for key representations that will be useful for all students.

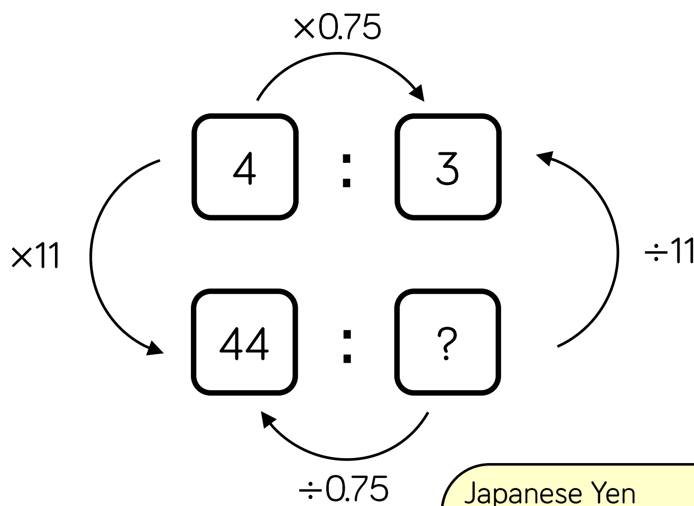
In many of the blocks of material, some of the small steps are in **bold**. These are content aimed at higher attaining students, but we would encourage teachers to use these with as many students as possible – if you feel your class can access any particular small step, then please include it in your planning.

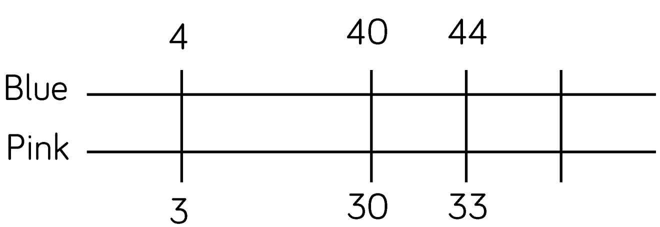
Key Representations

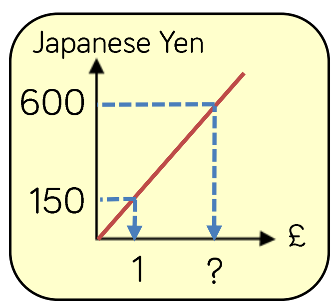












Concrete, pictorial and abstract representations are an important part of developing students' conceptual understanding.

Here are a few ideas on how you might represent ratio.

Bar models clearly show the equal parts, which section of the ratio has the largest share and help students to identify which part of the ratio they have been given, or are being asked to find.

Double number lines show the multiplicative nature of ratio and keep track of student's thinking.

Ratio and Scale

Small Steps

- Understand the meaning and representation of ratio
- Understand and use ratio notation
- Solve problems involving ratios of the form $1 : n$ (or $n : 1$)
- Solve proportional problems involving the ratio $m : n$
- Divide a value into a given ratio
- Express ratios in their simplest integer form
- Express ratios in the form $1 : n$ H
- Compare ratios and related fractions
- Understand π as the ratio between diameter and circumference
- Understand gradient of a line as a ratio H

H denotes higher strand and not necessarily content for Higher Tier GCSE

Representations of ratio

Notes and guidance

This small step is to ensure that students have a firmer understanding of the meaning of ratio. They should be able to represent ratios pictorially and have knowledge of the language involved. In pictorial representations, it is important to emphasise the equal parts of a ratio.

The link to fractions will be the focus of a later small step.

Key vocabulary

Ratio	Equal parts	For every
Proportion	Relationship	

Key questions

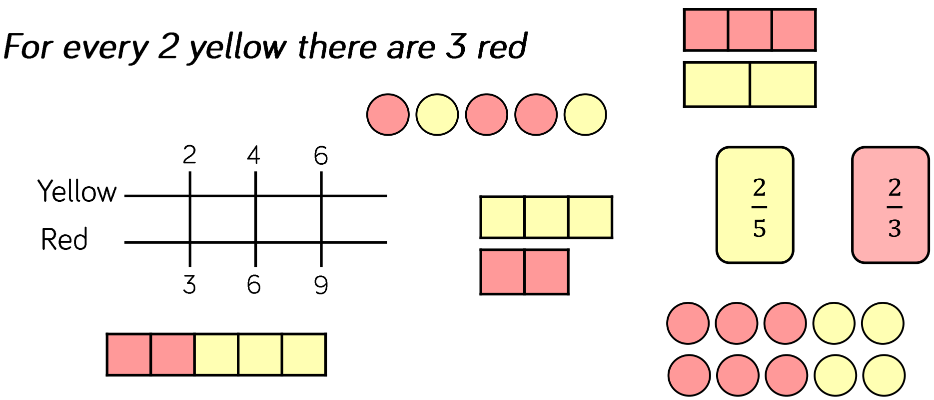
What is the purpose of a ratio?

Why should the blocks on a bar model be of equal size when representing ratio?

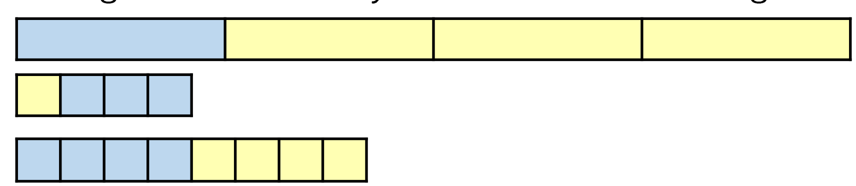
Exemplar Questions

What is the same and what is different about these representations?

For every 2 yellow there are 3 red



Green paint is made by mixing blue and yellow.
The more blue paint, the darker the green is. Which of these makes the darkest green? How could you make an even darker green?



How could you represent these phrases?

Faisal has twice as many cousins as Mo.	For every 50p you spend, we will donate an extra pound to charity.	1 in every 3 snakes is poisonous.

White Rose Maths

Understand and use ratio notation

Notes and guidance

This small step will introduce the use of the colon in ratio notation and link it to the representations explored in the previous step. The importance of order of terms within ratio notation should be highlighted. Most questions feature ratio comparing two parts, but students should be exposed to ratios involving multiple parts as well.

Key vocabulary

Equal parts	Order	Proportion
Ratio	Colon	

Key questions

Why is order important in ratio notation?

What does 1 : 1 mean?

Can ratio be used to compare more than two items?

Why are 2 : 1 and 1 : 2 different?

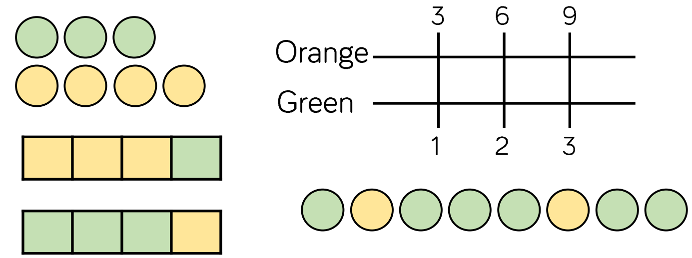
Exemplar Questions

Match each ratio card to its corresponding representation.

$3 : 1$

$3 : 4$

$1 : 3$



Strawberry Shake is made of strawberries, ice cream and milk in the ratio $1 : 2 : 5$. Write down the ratio of milk to ice cream. Write a suggestion for a new ratio to give a stronger strawberry flavour.

Write the ratios, and draw representations for $a : b$ when

$a = 3, b = 1$

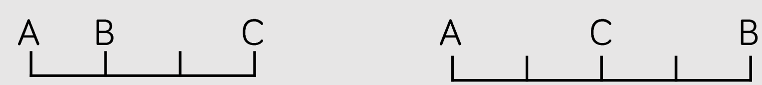
$a = 1, b = 3$

$a = 1, b = 1$

$a = b$

How would the ratios change if you added 1 to both a and b?
How would the ratios change if you doubled both a and b?

What is the ratio of the distance AB : BC in the following lines?



Can you position A,B and C on a line so that the ratio AB : BC is $2 : 5$? How many different ways can you do this?

Solve problems in the ratio $1 : n$

Notes and guidance

In this small step, students will use simple multiplicative reasoning with ratio. In this early stage of ratio learning, n will always be an integer.

For larger values of n, students can be introduced to the advantages of a double number line to support their calculations.

Key vocabulary

Divide	Proportional	Multiply
Part	Double number line	

Key questions

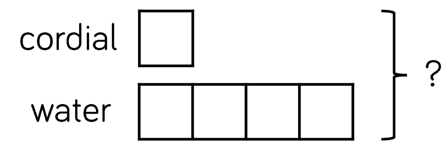
In the ratio $1 : n$, which is the bigger part?

Is bar modelling suitable for all ratios?

How are the ratios 1 : 20 and 20 : 1 different?

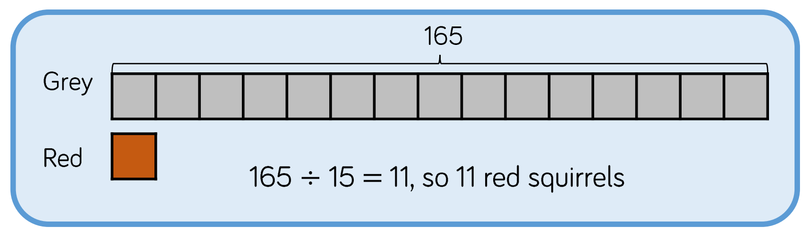
Exemplar Questions

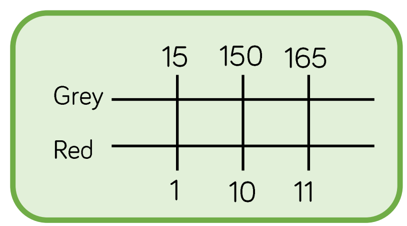
Juice is made using cordial and water in a ratio of $1 : 4$
Use the bar model to work out how much juice will be made with 40 ml of cordial.

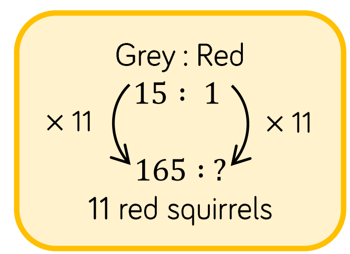


What if there were 40 ml of water?
What if there were 40 ml in total?

The ratio of grey to red squirrels in a forest is $15 : 1$
There are 165 grey squirrels. How many red squirrels are there?
Which of these representations do you prefer and why?







Solve problems in the ratio $m : n$

Notes and guidance

Students will be familiar with terminology of e.g. 'for every 4, there are 3'. from KS2. They will now develop their understanding of ratio alongside formal mathematical notation. Students can explore multiple methods including double number lines, finding multipliers or using bar models and then discuss which is most appropriate to the problem.

Key vocabulary

Proportional	Equal Parts	Multiplier
Placeholder	Units	

Key questions

Can there be more than two amounts in a ratio?

Does adding 1 to each part change the ratio?

How do you set up a bar model for a ratio like 3 : 2? Does the size of the bars matters?

Exemplar Questions

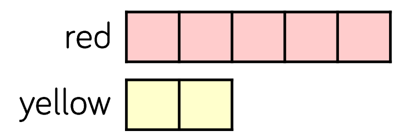
The ratio of men to women in the doctor's waiting room is $4 : 3$

Decide which of these are always, sometimes or never true:

- There are more men than women
- For every 3 men there are 4 women
- There are 7 men and women altogether
- If another man walks in the ratio will change to $5 : 3$

Can you draw a model to support your answers?

A shop orders red and yellow flowers in a ratio of $5 : 2$



One week they order 50 yellow flowers.
How many red flowers do they order?
The following week they order 50 red flowers.
How many yellow flowers do they order?
One week they order 140 flowers altogether.
How many more red flowers than yellow did they buy?

For every 2 hours Jane revises for, she gives herself 30 minutes free time. Which of these ratios represent this situation, and which don't?

$2 : 30$ | $2 : 0.5$ | $2 : \frac{1}{2}$ | $120 : 30$ | 2hrs : 30mins

Divide a value into a given ratio

Notes and guidance

In this small step it is important to expose students to the many combinations of 'sharing in a ratio' questions that can be asked, and not just when the total is given.

Bar modelling gives students a strategy to ensure that they have understood the information and can represent clearly what is known and what is unknown. Varying the ratios for a constant given total is useful.

Key vocabulary

Share	Total	Label
Parts	Ratio	

Key questions

What is the total number of parts?

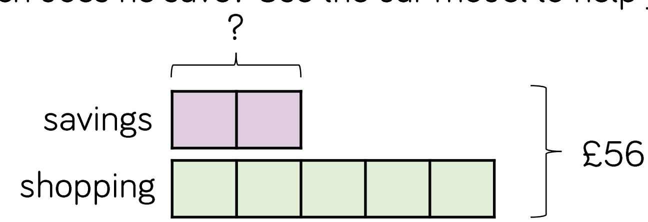
Where should you label the question mark in your bar model?

What other information does the bar model tell you?

Exemplar Questions

Sam decides to spend his monthly allowance of £56 on savings and shopping in the ratio $2 : 5$

How much does he save? Use the bar model to help you.



How much does he spend on shopping?

Next month he decides to save more, and changes the ratio to $3 : 5$

How much less does he spend on shopping this month?

What is the same and what is different about these questions?

Share £60 in the ratio of $1 : 5$

Share £120 in the ratio of $1 : 5$

Share £120 in the ratio of $2 : 10$

Share £120 in the ratio of $2 : 9 : 1$

A spice mix is made of cayenne pepper and paprika in the ratio 5 : 3
How much paprika is in 45.44 g of the mix?

Tom, Sam and Harry share some money in the ratio 2 : 3 : 5
How much does Tom get if the total is £60?
How much does Tom get if Sam gets £60?
How much does Tom get if Harry gets £60?

3 numbers in the ratio $2 : 3 : 7$ have a mean of 48
What is the median of the numbers?

Express ratios in simplest form

Notes and guidance

The concept of simplifying by finding factors will be familiar to students from work on equivalent fractions. Ratios will be simplified to their smallest integer terms. Pictorial or concrete representations should be used to support understanding of the concept. It may be useful to look at the answers to questions in previous steps and simplify these to see the original ratio is obtained.

Key vocabulary

Factors	Equivalent	Divide
Simplify	Common factors	

Key questions

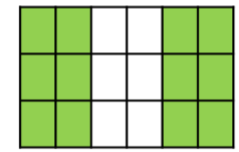
Why are factors useful when simplifying ratio?

What do we mean by 'common factors'?

When might you multiply to simplify a ratio?

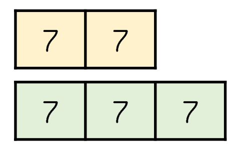
Exemplar Questions

Here is the flag of Nigeria. Explain how all these ratios could be used to describe the flag. Which is the most appropriate ratio and why?



Green : White	Green : White	Green : White
$2 : 1$	$6 : 3$	$12 : 6$

Explain how these show that the ratio $14 : 21$ can be simplified to $2 : 3$



$$14 : 21$$
$$7 \times 2 : 7 \times 3$$
$$2 : 3$$

$14 : 21$ ÷7 ÷7 $2 : 3$

Which of these ratios are the same?

$16 : 20$ | $8a : 10a$ | $4 : 5$ | $\frac{28}{45} : \frac{35}{45}$

$0.8 : 1$ | $4 \times 10^3 : 5 \times 10^2$

In a field there are 56 female sheep and 16 male sheep.
The farmer wants to keep the same ratio of female to male sheep in a field with only 14 female sheep.
How would simplifying the ratio help to answer this question?

White Rose Maths

Express ratios in the form $1 : n$ (H)

Notes and guidance

Here, students will use the simplification techniques from the previous small step to express a unit ratio; final ratios are no longer limited to integers.

It is easier to compare ratio in this format, and helps with later understanding of proportional change and scale factors.

Key vocabulary

Scale	Simplify	Compare
Divide	Units	

Key questions

If a ratio is simplified to the form $1 : n$, will n always be an integer?

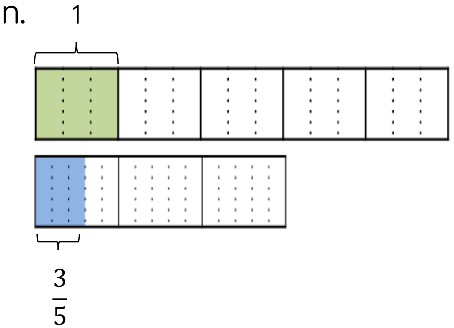
Why is the ratio format $1 : n$ useful for making comparisons?

Which would be larger, a $1 : 200$ scale model or a $1 : 300$ scale model?

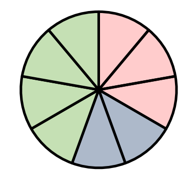
Exemplar Questions

To express the ratio $5 : 3$ in the form $1 : n$, explain how the bar model illustrates this simplification.

Divide each side by 5
5 : 3
5 ÷ 5 : 3 ÷ 5
1 : 0.6



The pie chart shows the colours of pixels in a logo.



Complete the sentences:

- For every one red pixel, there are … green pixels.
- For every one green pixel, there are … red pixels.

The ratio of which 2 colours would give the smallest value of n in the ratio $1 : n$?

---

11 : 2   4 : 9   4 : 6   6 : 4

5 : 2   5 : 8   4 kg : 200 g

Write these ratios in the form $1 : n$

What do your answers tell you about the different ratios?

Now write the ratios in the form $n : 1$

What's the same and what's different?

Compare ratios and fractions

Notes and guidance

The previous small steps highlighted total number of parts in a ratio, which is looked at again here when finding each part as a fraction of the whole. Students often incorrectly think e.g. the ratio 2 : 3 represents two-thirds of the whole.

Pictorial support or using cubes etc. is helpful here to address this misconception.

Key vocabulary

Total parts	Fraction	Proportion
Denominator	Numerator	

Key questions

What is the same and what is different when we look at a ratio and a fraction?

What's the connection between the sum of the parts of a ratio and its corresponding fraction?

Exemplar Questions

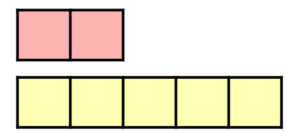
Match each statement to the bar model. How would you model the unmatched statements? What fractions can you see?

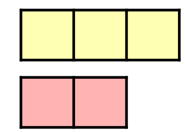
Two fifths are red

Yellow : Red 2 : 3

Two sevenths are yellow

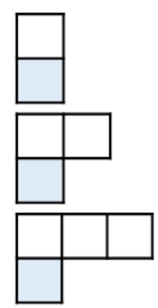
Red : Yellow 2 : 5





There are twice as many sheep to cows in a field.
What fraction of the animals in the field are sheep?
If there were 3 times as many sheep as cows, what fraction of the animals are cows?
If there were the same number of sheep as cows, what fraction of the animals are sheep?

John is writing ratios as fractions. Which column is wrong? Correct and continue his pattern.



B : W	Fraction Blue	W : B	Fraction White
1 : 1	$\frac{1}{2}$	1 : 1	$\frac{2}{1}$
1 : 2	$\frac{1}{3}$	2 : 1	$\frac{3}{1}$
1 : 3	$\frac{1}{4}$	3 : 1	$\frac{4}{1}$

Understand π as a ratio

Notes and guidance

Measuring circumferences and diameters of circular objects helps to establish that the circumference is a multiple of the diameter and to find an approximation for π. Defining π as the ratio of the circumference to the diameter leads to $\pi = \frac{C}{d}$ and then the formula for the circumference.

Students should then practise using this given the diameter or radius of circles, semi-circles etc.

Key vocabulary

Perimeter	Circumference	Constant
Pi (π)	Regular	Diameter

Key questions

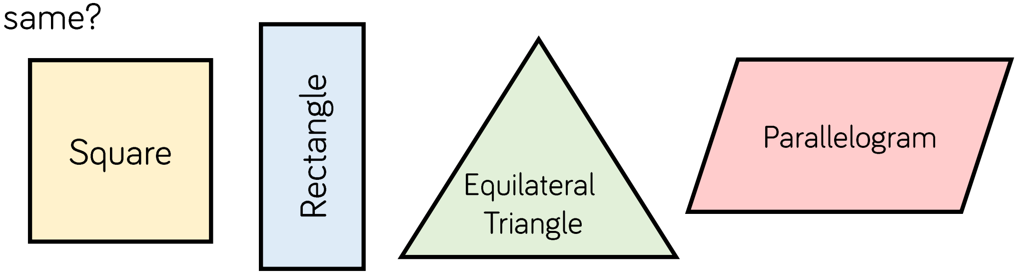
How is a square a rectangle? What makes it unique? Can you explain using ratio?

What's the difference between the radius of a circle and its diameter?

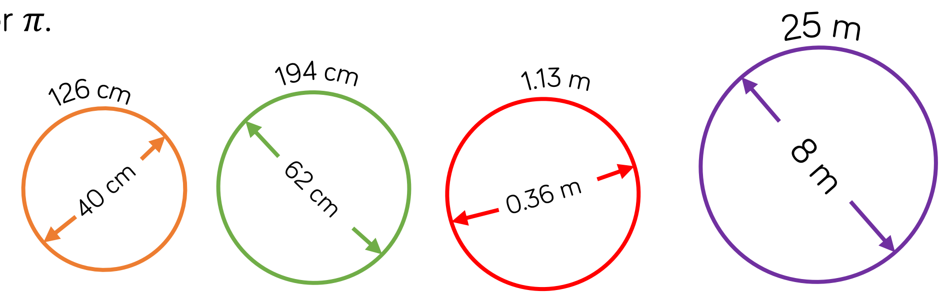
If I triple the diameter of a circle, what happens to its circumference?

Exemplar Questions

Explore the ratio of Width : Perimeter in the form $1 : n$ of the following shapes. For which shapes will the ratios always be the same?



The ratio of diameter : circumference in the form $1 : n$ of a circle is constant. It is $1 : \pi$. Use the circles given to find an approximation for π.



Which of these formulae are correct?

$C = \pi d$ $\quad d = \pi C$ $\quad C = 2\pi r$ $\quad \frac{C}{d} = \pi$

Calculate the circumference of a circle with diameter 0.4 cm
Calculate the perimeter of a semicircle with a diameter 0.4 cm

Understand gradient as a ratio H

Notes and guidance

In this small step, students will formally discuss gradient for the first time, having considered slope only briefly in Year 7 Making the link between gradient and ratio will help deepen understanding and the making of links between many areas of the curriculum, including scale factors, direct proportion graphs etc. At this stage, only positive gradients are considered.

Key vocabulary

Right-angled triangle	Gradient
Slope	Steep

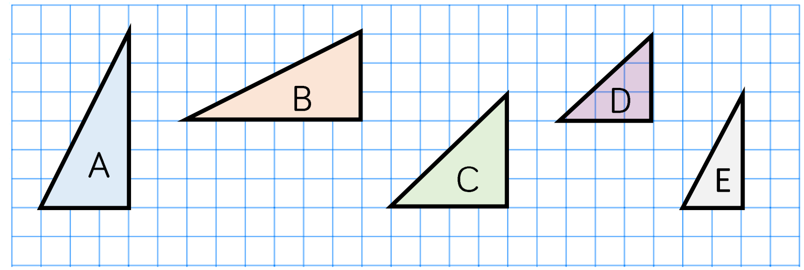
Key questions

What does gradient measure?

What happens to the gradient as a line gets steeper?

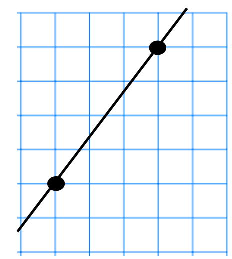
How is the gradient of $\frac{3}{4}$ different to a gradient of $\frac{4}{3}$?

Exemplar Questions



Which of these triangles have the same height to width ratio?
Write the width : height ratio of each triangle in the form $1 : n$
Draw the triangles in ascending order of n. What do you notice?
Draw 3 different triangles with a width to height ratio of 1 : 3

The gradient of this line is $\frac{4}{3}$
Use a triangle and the points on the line to explain why.
How is this different to a gradient of $\frac{3}{4}$?



A line has a gradient of 3. One of the points on the line is (4,2)

Natasha says "to work out another point on the line, add 3 to the y-coordinate for every one that you add to the x-coordinate".

Draw a diagram to see if she is correct. How many points on the line could you find?